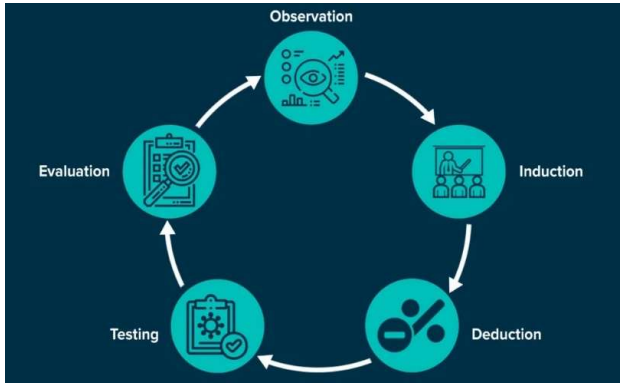
Empirical research in management and economics

Factor analysis



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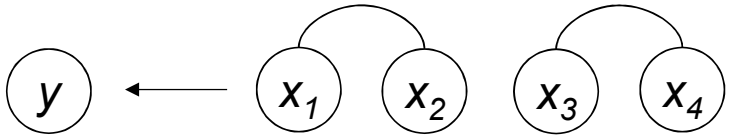
Recap from last week

- In logistic regression, what does the linear combination of intercept and predictors predict?
- How are probability, odds, and log odds related to each other?
- How do you get from an estimated slope (i.e., regression coefficient) of a predictor to the odds ratio?
- How do you test whether an estimated regression coefficient differs significantly from zero?
- How can you assess the performance of a logistic regression model—with and without taking model complexity into account?
- What are key assumptions in logistic regression?
- How can you plan the sample size for a logistic regression analysis?

Agenda for the semester

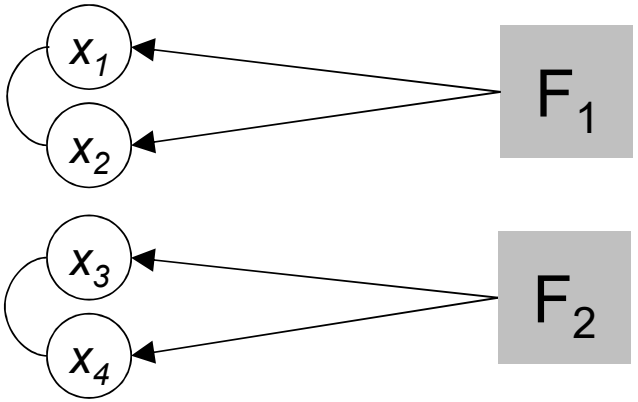
Session	Date	Topic
1	13 October	Introduction
2	20 October	Descriptive data analysis
3	27 October	Hypothesis development and measurement
4	3 November	Inferential data analysis I
5	10 November	Inferential data analysis II
6	17 November	Simple regression
7	24 November	Multiple regression
8	1 December	Logistic regression
9	**8 December**	**Factor analysis**
10	15 December	Cluster analysis
11	12 January	Conjoint analysis
12	19 January	The replication crisis and open science
13	26 January	Summary and questions
	11 February	Exam

Regression analysis



observed observed

Factor-analytic techniques



observed latent

Example: Decision styles

M = 21 variables (items)

N = 149 objects (persons)

Responses on a 5-point scale ranging from 1 = “I don’t agree” to 5 = “I agree completely.”

Item no	Item formulation
1	Developing a clear plan is very important to me.
2	When I make a decision, it is more important for me to feel the decision is right than to have a rational reason for it.
3	I make quick decisions.
4	I usually have clear, explainable reasons for my decisions.
5	I prefer drawing conclusions based on my feelings, my knowledge of human nature, and my experience of life.
6	I quickly do the right thing when deciding because I’ve often faced almost the same thing before.
7	Before making decisions I usually think about the goals I want to achieve.
8	I hardly ever go wrong when I listen to my deepest gut feelings to find an answer.
9	I rarely need to mull things over; how to decide usually becomes quickly apparent.
10	I prefer making detailed plans rather than leaving things to chance.
11	When I make a decision, I trust my inner feelings and reactions.
12	I’ve had enough experience to just know what I need to do most of the time without trying to figure it out every time.
13	When I make decisions, I proceed step-by-step.
14	With most decisions it makes sense to completely rely on your feelings.
15	I typically figure out the way to decide swiftly.
16	Using my gut feelings usually works well for me in figuring out problems in my life.
17	I like detailed action plans.
18	I am often aware of how to decide even before I review all aspects.
19	I believe in trusting my hunches.
20	The right way to decide usually comes to mind almost immediately.
21	I tend to use my heart as a guide for my actions.

→ What is the (small) set of dimensions underlying people‘s responses to these items?

Pachur & Spaar (2015)

Dimensionality reduction

Variables

Factors

Goals for this week

- You understand the goal of **dimensionality reduction**
- You are familiar with key principles and elements of **principle component analysis**: Factors (components), factor loadings
- You understand approaches to decide how many of the extracted factors to **retain** (based on *eigenvalues* of factors)
- You know the notion of **factor rotation** and different types of rotation
- You have some experience with **interpreting** extracted factors
- You understand what **factor scores** are
- You know some **preparatory and practical considerations** when conducting a factor analysis

Factor-analytic techniques

- Purposes
  - Discover the factors that underlie the covariation among multiple observed variables
  - Reduce a large set of variables into a smaller set of factors
- Types of techniques
  - Factor analysis
    → Estimation of latent factors and generalization to a population
    - *Exploratory* factor analysis
    - *Confirmatory* factor analysis
  - (Exploratory) Principle component analysis (PCA)
    → Summary of the correlational structure in a given dataset

Example: Decision styles

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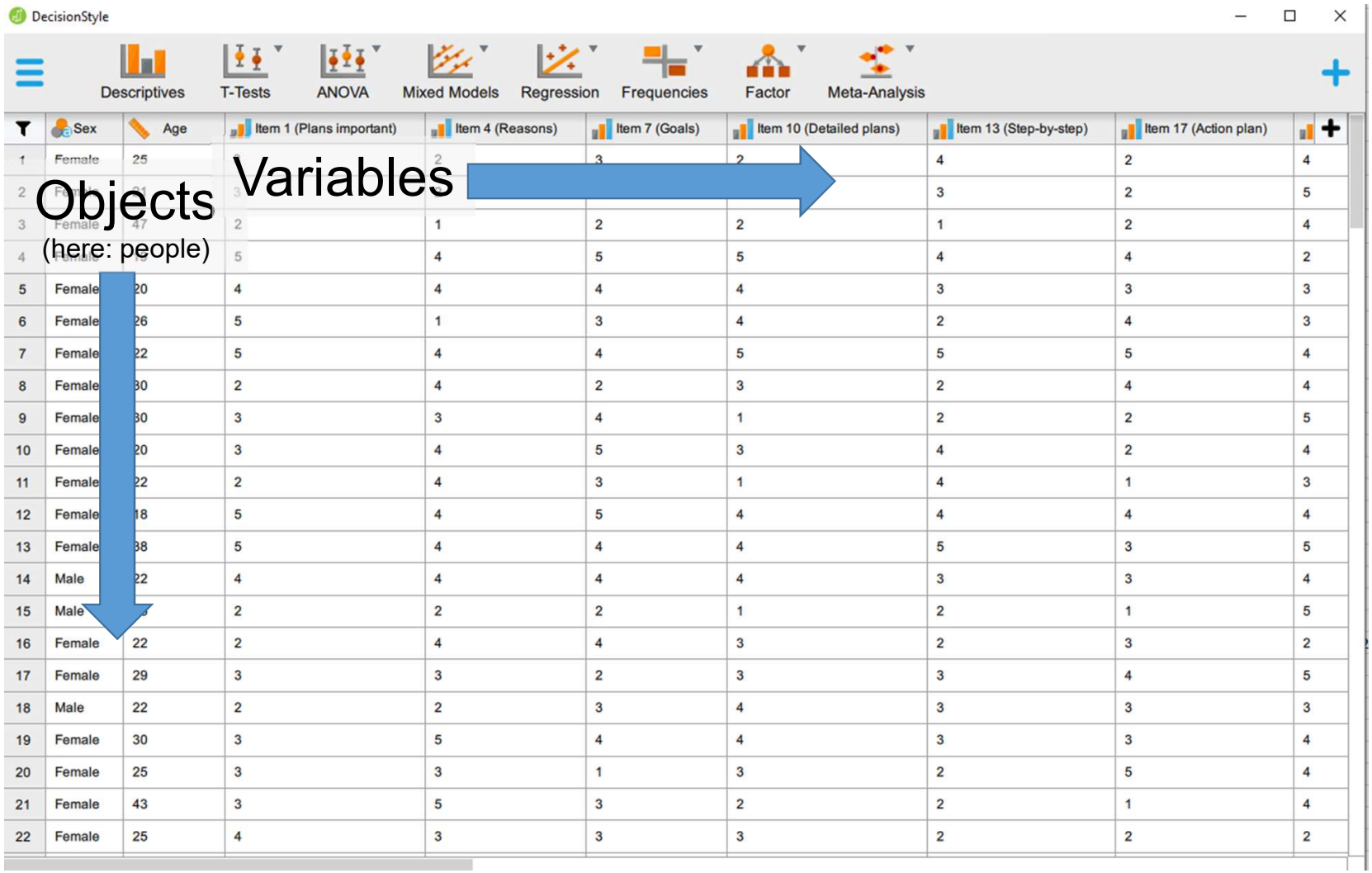
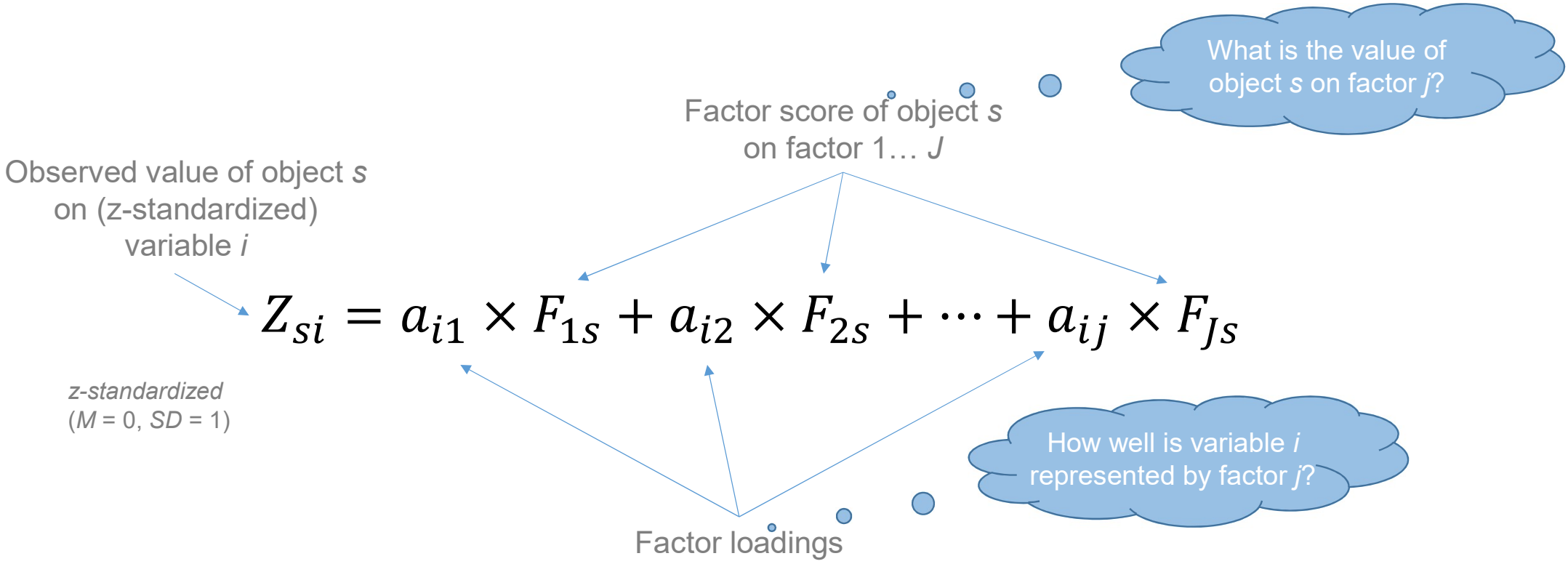
Pachur & Spaar (2015)



	Sex	Age	Item 1 (Plans important)	Item 4 (Reasons)	Item 7 (Goals)	Item 10 (Detailed plans)	Item 13 (Step-by-step)	Item 17 (Action plan)	
1	Female	25		2	3	2	4	2	4
2							3	2	5
3	Female	47	2	1	2	2	1	2	4
4			5	4	5	5	4	4	2
5	Female	20	4	4	4	4	3	3	3
6	Female	26	5	1	3	4	2	4	3
7	Female	22	5	4	4	5	5	5	4
8	Female	30	2	4	2	3	2	4	4
9	Female	30	3	3	4	1	2	2	5
10	Female	20	3	4	5	3	4	2	4
11	Female	22	2	4	3	1	4	1	3
12	Female	18	5	4	5	4	4	4	4
13	Female	38	5	4	4	4	5	3	5
14	Male	22	4	4	4	4	3	3	4
15	Male		2	2	2	1	2	1	5
16	Female	22	2	4	4	3	2	3	2
17	Female	29	3	3	2	3	3	4	5
18	Male	22	2	2	3	4	3	3	3
19	Female	30	3	5	4	4	3	3	4
20	Female	25	3	3	1	3	2	5	4
21	Female	43	3	5	3	2	2	1	4
22	Female	25	4	3	3	3	2	2	2

Principle component analysis: The model



Correlation matrix

	Item 1 (Plans important)	Item 4 (Reasons)	Item 7 (Goals)	Item 10 (Detailed plans)	Item 13 (Step-by-step)	Item 17 (Action plan)	Item 2 (Feel right)	Item 3 (Quick)	Item 5 (Human nature)	Item 6 (Quickly)	Item 8 (Deep feelings)	Item 9 (No mulling)	Item 11 (Inner reactions)	Item 12 (Experience)	Item 14 (Feelings)	Item 15 (Swiftly)	Item 16 (Gut feelings)	Item 18 (Aware)	Item 19 (Hunch)	Item 20 (Come to mind)
Item 1 (Plans important)	—																			
Item 4 (Reasons)	0.32	—																		
Item 7 (Goals)	0.36	0.43	—																	
Item 10 (Detailed plans)	0.65	0.39	0.44	—																
Item 13 (Step-by-step)	0.38	0.21	0.38	0.44	—															
Item 17 (Action plan)	0.59	0.26	0.30	0.62	0.47	—														
Item 2 (Feel right)	-0.14	-0.17	-0.08	-0.27	-0.16	-0.16	—													
Item 3 (Quick)	0.12	0.16	-0.02	0.08	-0.07	0.09	0.12	—												
Item 5 (Human nature)	-0.18	-0.17	-0.10	-0.24	-0.15	-0.13	0.55	0.10	—											
Item 6 (Quickly)	0.13	0.10	0.01	0.14	0.09	0.18	0.04	0.43	0.09	—										
Item 8 (Deep feelings)	-0.01	0.04	-0.01	0.01	0.08	0.06	0.22	0.03	0.41	0.22	—									
Item 9 (No mulling)	0.03	0.04	-0.06	-0.04	0.03	-0.01	0.18	0.46	0.18	0.41	0.22	—								
Item 11 (Inner reactions)	-0.21	-0.10	-0.13	-0.26	-0.19	-0.28	0.56	0.11	0.53	0.05	0.35	0.19	—							
Item 12 (Experience)	0.12	0.25	0.19	0.14	0.14	0.13	0.03	0.35	0.14	0.55	0.28	0.47	0.14	—						
Item 14 (Feelings)	-0.21	-0.149	-0.14	-0.20	-0.04	-0.09	0.55	0.14	0.40	0.07	0.36	0.32	0.51	0.18	—					
Item 15 (Swiftly)	0.17	0.16	0.07	0.06	0.02	0.06	0.03	0.48	0.01	0.47	0.23	0.58	0.07	0.58	0.22	—				
Item 16 (Gut feelings)	-0.095	-0.05	0.01	-0.26	-0.01	-0.05	0.43	0.17	0.41	0.14	0.46	0.28	0.41	0.24	0.51	0.34	—			
Item 18 (Aware)	0.14	0.19	0.01	0.23	0.04	0.09	-0.03	0.23	0.10	0.32	0.14	0.33	0.03	0.27	0.04	0.33	0.10	—		
Item 19 (Hunch)	-0.17	0.01	-0.05	-0.16	-0.13	-0.14	0.21	0.10	0.29	0.15	0.35	0.22	0.34	0.33	0.32	0.24	0.35	0.30	—	
Item 20 (Come to mind)	0.04	0.13	0.10	0.14	0.14	0.08	0.03	0.41	0.07	0.46	0.21	0.50	0.10	0.51	0.23	0.54	0.19	0.43	0.42	—
Item 21 (Heart)	-0.13	-0.16	-0.04	-0.25	-0.13	-0.18	0.57	0.21	0.48	0.09	0.33	0.26	0.51	0.26	0.69	0.27	0.54	0.10	0.39	0.19

$\rightarrow R$

Eigenvalue matrix

Matrix **R** is *diagonalized* to obtain the *eigenvalue matrix* **L**

$$\boldsymbol{L} = \boldsymbol{V}' \cdot \boldsymbol{R} \cdot \boldsymbol{V}$$

Eigenvalue matrix

Eigenvalue matrix

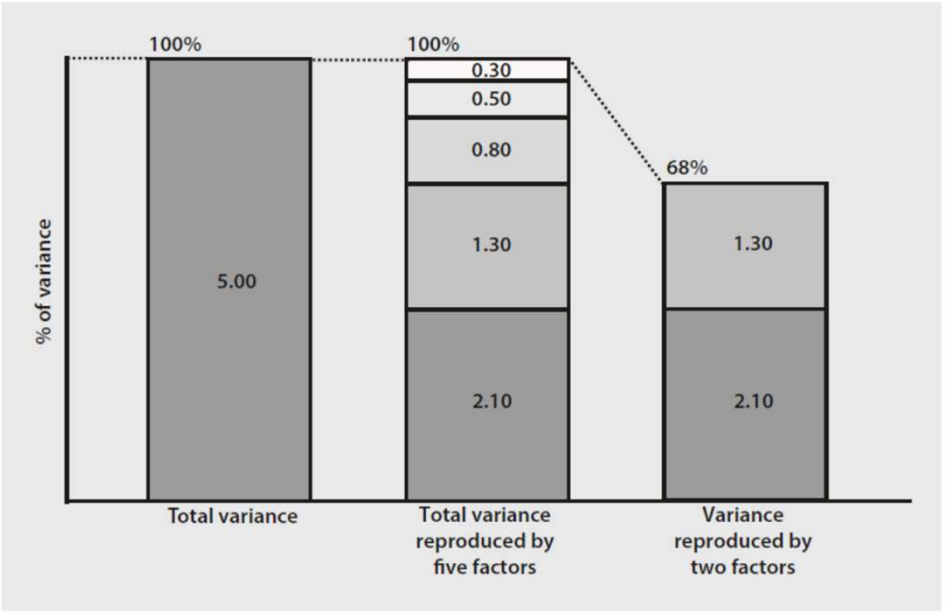
Eigenvalue: Sum of the variance of the variables that is reproduced by a given factor

Assuming there are 5 variables

$$\boldsymbol{L} = \begin{bmatrix} 2.1 & 0 & 0 & 0 & 0 \\ 0 & 1.3 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

Eigenvalues

→ Initially, 5 factors are extracted

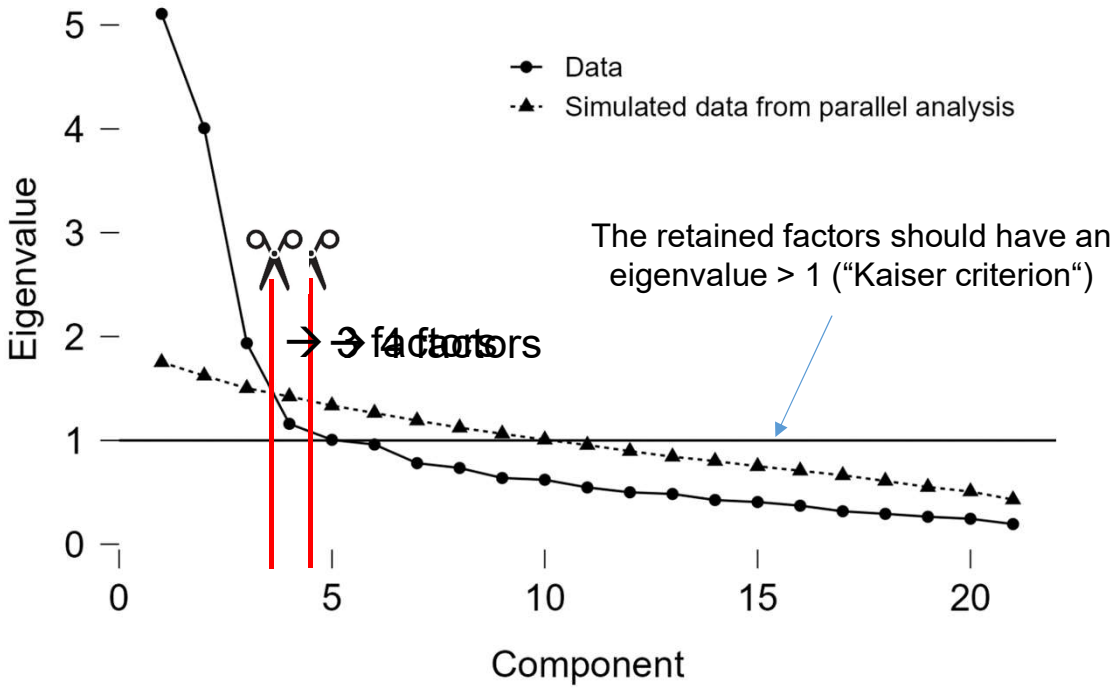


L has the eigenvalues in the positive diagonal and zeros in the other cells

How many factors should be retained?

Scree test (Cattell, 1966)

- Eigenvalues of the factors are plotted in descending order (*scree plot*)
- Determine the point in plot where the distribution of eigenvalues levels off → "Elbow criterion"



How many factors should be retained?

Possible approaches

- Kaiser criterion

  The retained components should have an eigenvalue > 1

- Scree test (Cattell, 1966)

  Plot eigenvalues of the factors in descending order (*scree plot*), determine the point where the distribution of eigenvalues levels off (→ "Elbow criterion") and keep the factors above this point

- Parallel analysis (recommended)

  → Based on simulations, it provides a benchmark for each factor in terms of the eigenvalue expected by chance

  Procedure

  - Random datasets are generated many times (with the same number of variables and objects as in the empirical dataset)
  - For each dataset a PCA is conducted
  - For each extracted factor the average (across repetitions) eigenvalue is plotted → eigenvalue expected by chance
  - Retain the factors whose eigenvalue is larger than the respective eigenvalue determined by the simulation

From correlation matrix to factor loadings matrix

$$\boldsymbol{L = V' \cdot R \cdot V}$$

Eigenvalue matrix

$$\boldsymbol{A = V \cdot \sqrt{L}}$$

Factor loadings matrix

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{I1} & \cdots & a_{IJ} \end{bmatrix}$$

Factors

Variables

Factor loadings

Factor loadings

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{I1} & \cdots & a_{IJ} \end{bmatrix}$$

Factor loadings matrix (*pattern matrix*)

Component Loadings (*a*)

	PC1	PC2	PC3	Uniqueness
Item 21 (Heart)	0.742	−0.207	0.281	0.327
Item 14 (Feelings)	0.705	−0.206	0.300	0.371
Item 16 (Gut feelings)	0.681	−0.082	0.285	0.448
Item 11 (Inner reactions)	0.633	−0.328	0.268	0.421
Item 9 (No mulling)	0.611	0.323	−0.290	0.439
Item 19 (Hunch)	0.594	−0.005	−0.041	0.645
Item 2 (Feel right)	0.582	−0.346	0.383	0.395
Item 5 (Human nature)	0.570	−0.290	0.271	0.518
Item 15 (Swiftly)	0.559	0.480	−0.325	0.351
Item 12 (Experience)	0.539	0.524	−0.148	0.413
Item 20 (Come to mind)	0.534	0.485	−0.285	0.398
Item 8 (Deep feelings)	0.507	0.088	0.288	0.652
Item 3 (Quick)	0.435	0.374	−0.309	0.575
Item 6 (Quickly)	0.416	0.491	−0.302	0.494
Item 18 (Aware)	0.328	0.420	−0.218	0.668
Item 10 (Detailed plans)	−0.272	0.731	0.317	0.291
Item 1 (Plans important)	−0.199	0.655	0.361	0.401
Item 17 (Action plan)	−0.185	0.638	0.403	0.396
Item 13 (Step-by-step)	−0.145	0.522	0.426	0.525
Item 7 (Goals)	−0.126	0.490	0.433	0.556
Item 4 (Reasons)	−0.075	0.556	0.139	0.666

Note. No rotation method applied.

Communality

$$h_i^2 = \sum_{j=1}^{J} a_{ij}^2$$

Factor loading (***a***): Correlation of a variable with a factor

Communality (h^2): Proportion of variance of a variable accounted for by the extracted factors

Uniqueness ≈ 1 – communality: Proportion of variance of a variable **not** accounted for by the extracted factors

Component Characteristics

	Eigenvalue	Proportion var.	Cumulative
Component 1	5.108	0.243	0.243
Component 2	4.007	0.191	0.434
Component 3	1.936	0.092	0.526

Eigenvalue

$$\lambda_j = \sum_{i=1}^{I} a_{ij}^2$$

Factor loadings

Factor loadings matrix (*pattern matrix*)

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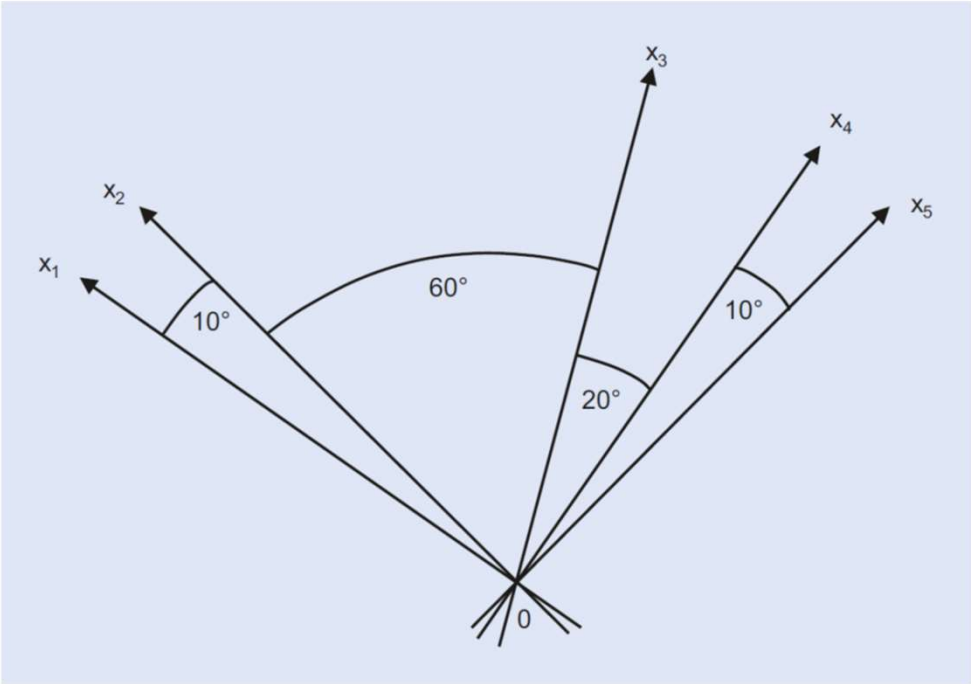
Note. No rotation method applied.

Substantial loadings on multiple factors → Complicates interpretation of the factors (i.e., what latent content does each factor represent?)

To facilitate interpretation, it is therefore useful to aim for **simple structure** (i.e., have a high loading on one factor, low loadings on all other factors)

Geometric representation

Correlation between variables as the angle between vectors



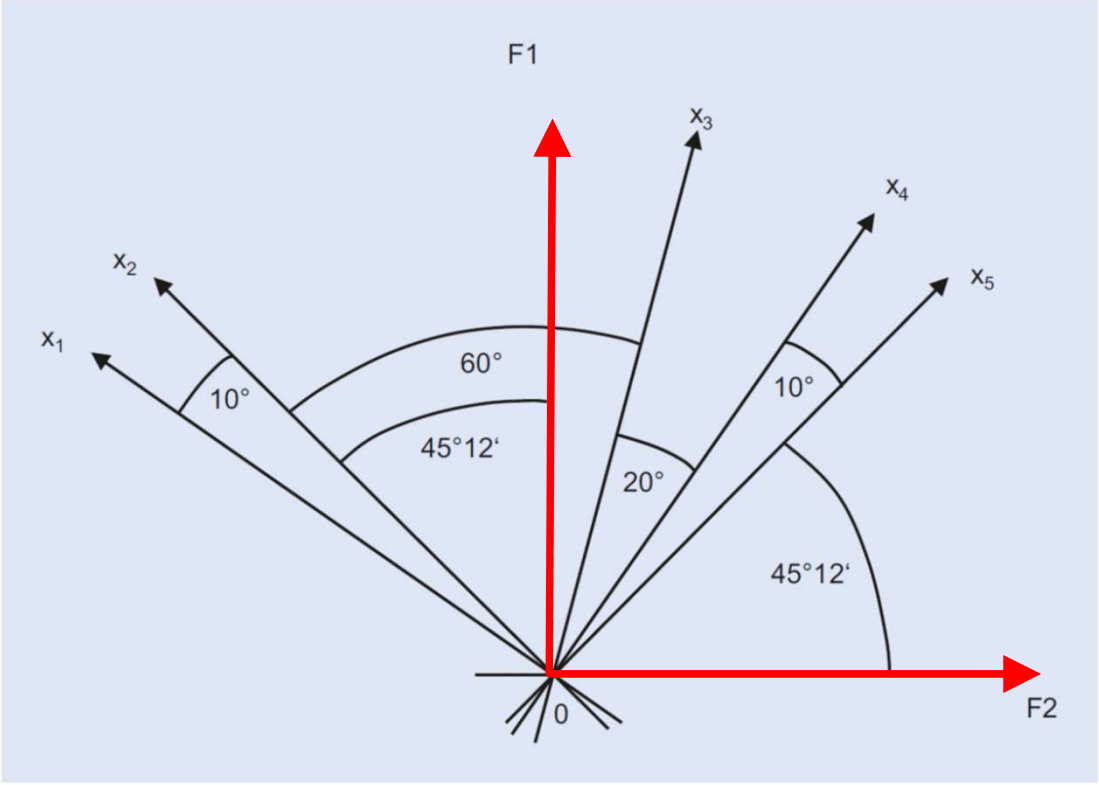
$$r_{x_i x_j} = \cos \theta_{x_i x_j}$$

e.g. $r_{x_i x_j} = \cos 90° = 0$

	x_1	x_2	x_3	x_4	x_5
x_1	1	10°	70°	90°	100°
x_2	0,985	1	60°	80°	90°
x_3	0,342	0,500	1	20°	30°
x_4	0,000	0,174	0,940	1	10°
x_5	−0,174	0,000	0,866	0,985	1

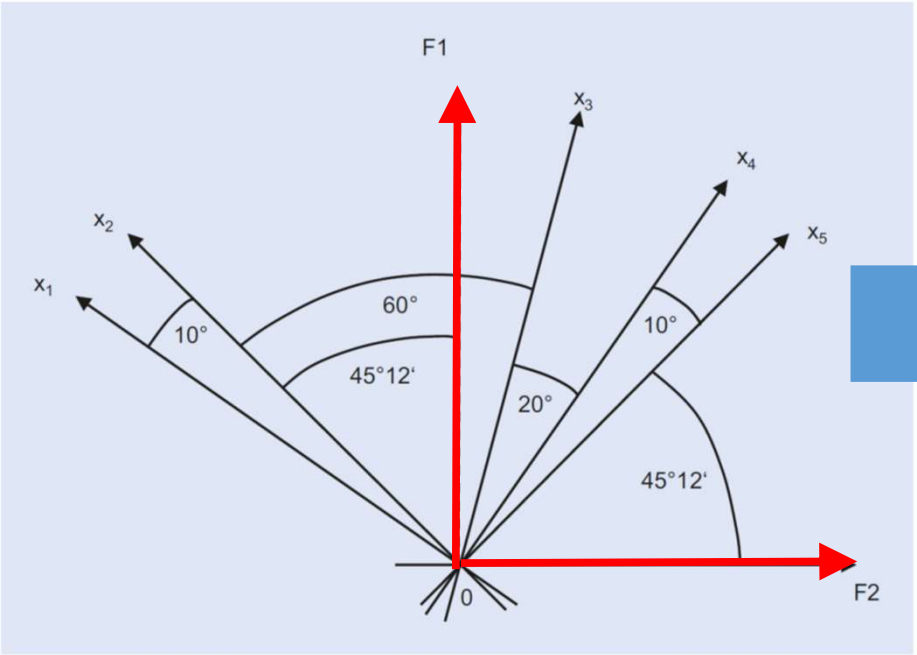
Correlations (*r*)

Geometric representation

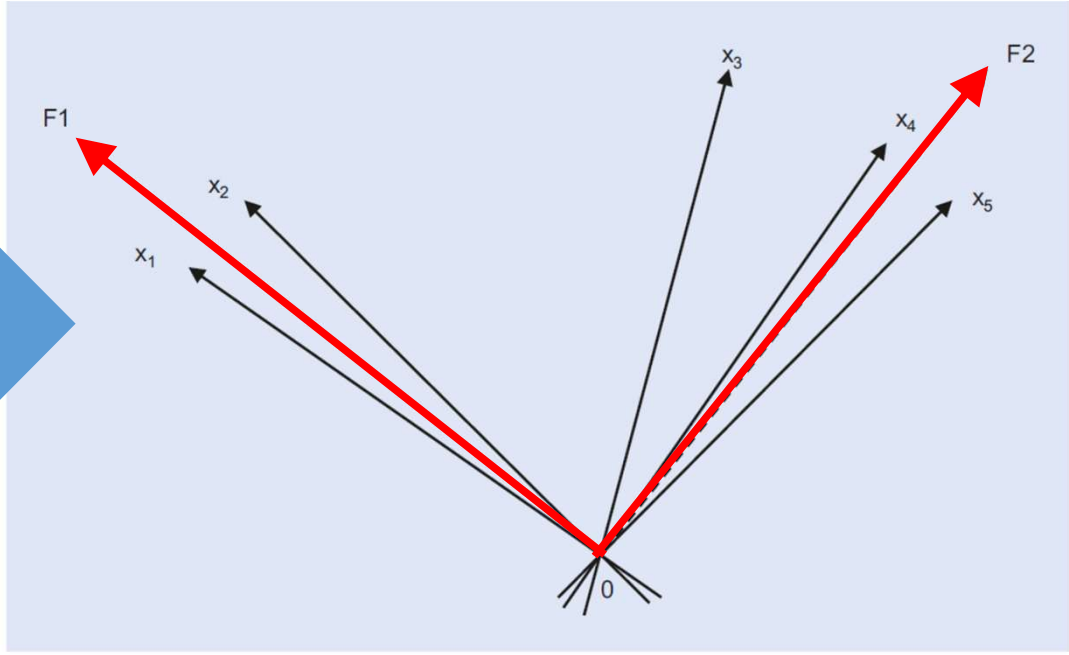


Facilitating interpretation: Rotation

Original solution

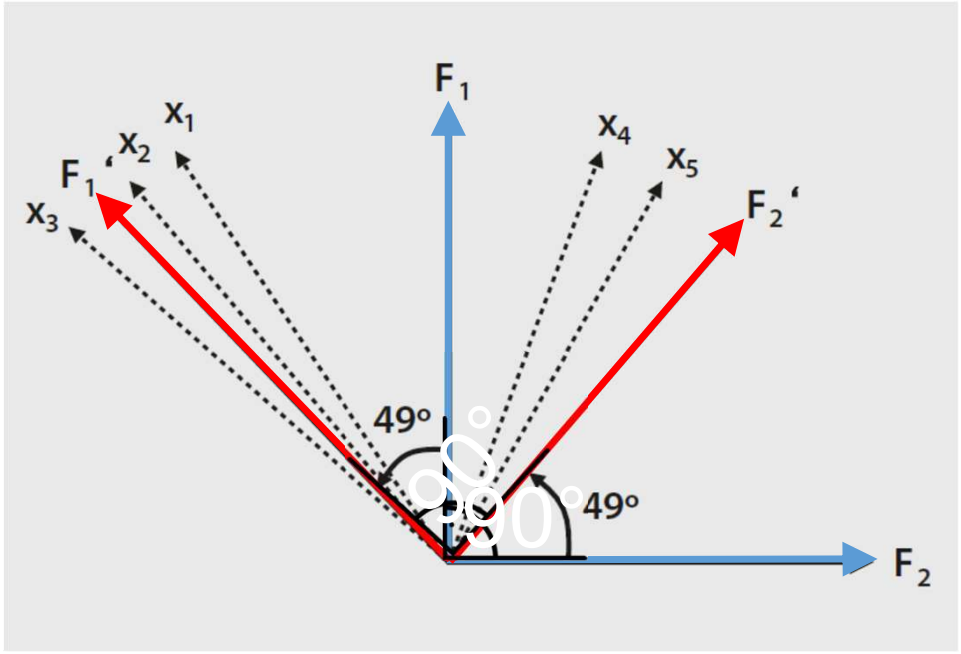


Rotated solution



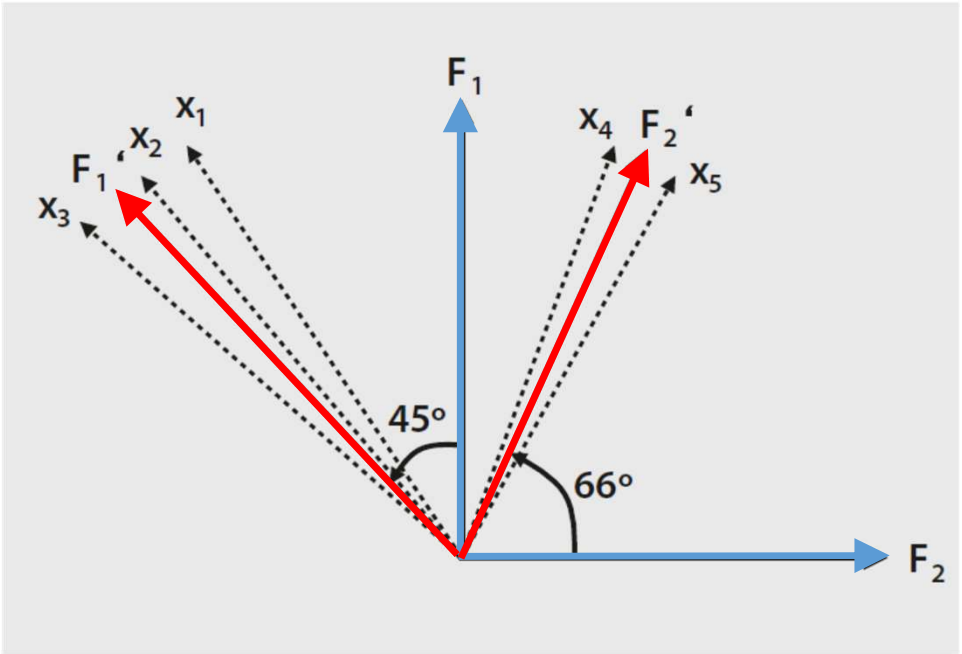
Facilitating interpretation: Rotation

Orthogonal rotation



→ Rotated factors are uncorrelated (i.e., 90°)

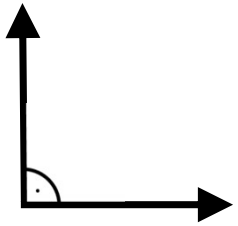
Oblique rotation



→ Rotated factors can be correlated (i.e., ≠ 90°)

Factor loading: Rotation procedures

Varimax: Maximize variance of squared factor loadings across ***variables***



Orthogonal rotation

Quartimax: Maximize variance of squared factor loadings across ***factors***

Component Loadings

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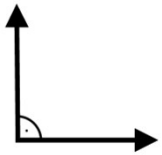
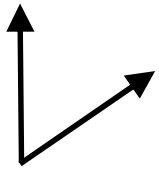
Note. No rotation method applied.

Facilitating interpretation: Rotation

Common procedures for rotation



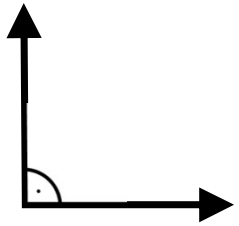
- Orthogonal
  - Varimax: ***Max***imizes ***vari***ance of squared factor loadings across the variables
  - Quartimax: Maximizes variance of squared factor loadings across the factors



- Oblique
  - Oblimin: Minimize cross-products of loadings
  - Promax (recommended): Based on varimax but operates on exponentiated factor loadings, which accentuates loading patterns (it is computationally faster than oblimin)

Orthogonal rotation

Factor loadings matrix for rotated factors (*structure matrix*)



Component Loadings

	PC1	PC2	PC3	Uniqueness
Item 21 (Heart)	0.794			0.327
Item 14 (Feelings)	0.774			0.371
Item 2 (Feel right)	0.764			0.395
Item 11 (Inner reactions)	0.734			0.421
Item 16 (Gut feelings)	0.714			0.448
Item 5 (Human nature)	0.676			0.518
Item 8 (Deep feelings)	0.533			0.652
Item 19 (Hunch)	0.441			0.645
Item 15 (Swiftly)		0.795		0.351
Item 20 (Come to mind)		0.762		0.398
Item 12 (Experience)		0.715		0.413
Item 9 (No mulling)		0.713		0.439
Item 6 (Quickly)		0.704		0.494
Item 3 (Quick)		0.649		0.575
Item 18 (Aware)		0.564		0.668
Item 10 (Detailed plans)			0.802	0.291
Item 17 (Action plan)			0.769	0.396
Item 1 (Plans important)			0.759	0.401
Item 13 (Step-by-step)			0.689	0.525
Item 7 (Goals)			0.666	0.556
Item 4 (Reasons)			0.520	0.666

Note. Applied rotation method is varimax.

Unrotated solution

Component Loadings

	PC1	PC2	PC3	Uniqueness
Item 21 (Heart)	0.742			0.327
Item 14 (Feelings)	0.705			0.371
Item 16 (Gut feelings)	0.681			0.448
Item 11 (Inner reactions)	0.633			0.421
Item 9 (No mulling)	0.611			0.439
Item 19 (Hunch)	0.594			0.645
Item 2 (Feel right)	0.582			0.395
Item 5 (Human nature)	0.570			0.518
Item 15 (Swiftly)	0.559	0.480		0.351
Item 12 (Experience)	0.539	0.524		0.413
Item 20 (Come to mind)	0.534	0.485		0.398
Item 8 (Deep feelings)	0.507			0.652
Item 3 (Quick)	0.435			0.575
Item 6 (Quickly)	0.416	0.491		0.494
Item 10 (Detailed plans)		0.731		0.291
Item 1 (Plans important)		0.655		0.401
Item 17 (Action plan)		0.638	0.403	0.396
Item 4 (Reasons)		0.556		0.666
Item 13 (Step-by-step)		0.522	0.426	0.525
Item 7 (Goals)		0.490	0.433	0.556
Item 18 (Aware)		0.420		0.668

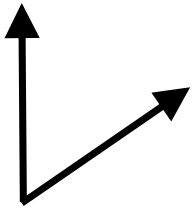
Note. No rotation method applied.

Component Characteristics

	Unrotated solution			Rotated solution		
	Eigenvalue	Proportion var.	Cumulative	SumSq. Loadings	Proportion var.	Cumulative
Component 1	5.108	0.243	0.243	4.018	0.191	0.191
Component 2	4.007	0.191	0.434	3.809	0.181	0.373
Component 3	1.936	0.092	0.526	3.223	0.153	0.526

Oblique rotation

Factor loadings matrix for rotated factors (*structure matrix*)



Component Loadings ▼

	RC1	RC2	RC3	Uniqueness
Item 2 (Feel right)	0.806			0.395
Item 21 (Heart)	0.792			0.327
Item 14 (Feelings)	0.781			0.371
Item 11 (Inner reactions)	0.743			0.421
Item 16 (Gut feelings)	0.716			0.448
Item 5 (Human nature)	0.691			0.518
Item 8 (Deep feelings)	0.549			0.652
Item 15 (Swiftly)		0.817		0.351
Item 20 (Come to mind)		0.777		0.398
Item 6 (Quickly)		0.731		0.494
Item 9 (No mulling)		0.726		0.439
Item 12 (Experience)		0.702		0.413
Item 3 (Quick)		0.677		0.575
Item 18 (Aware)		0.581		0.668
Item 10 (Detailed plans)			0.789	0.291
Item 17 (Action plan)			0.782	0.396
Item 1 (Plans important)			0.762	0.401
Item 13 (Step-by-step)			0.717	0.525
Item 7 (Goals)			0.698	0.556
Item 4 (Reasons)			0.496	0.666
Item 19 (Hunch)				0.645

Note. Applied rotation method is promax.

→ How are the extracted factors correlated?

Component Correlations ▼

	Component 1	Component 2	Component 3
Component 1	1.000	0.292	−0.226
Component 2	0.292	1.000	0.168
Component 3	−0.226	0.168	1.000

Component Characteristics

	Unrotated solution			Rotated solution		
	Eigenvalue	Proportion var.	Cumulative	SumSq. Loadings	Proportion var.	Cumulative
Component 1	5.108	0.243	0.243	3.987	0.190	0.190
Component 2	4.007	0.191	0.434	3.855	0.184	0.373
Component 3	1.936	0.092	0.526	3.208	0.153	0.526

Interpreting the factors

- Based on which variables load most highly on a given factor
- Recommenation: only consider variables with factor loadings > |.4| (Stevens, 2002)
- Involves a certain degree of subjectivity

Component Loadings ▼

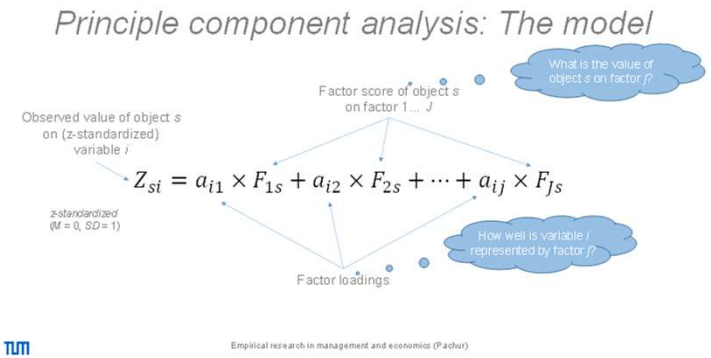
	RC1	RC2	RC3
Item 2 (Feel right)	0.806		
Item 21 (Heart)	0.792		
Item 14 (Feelings)	0.781		
Item 11 (Inner reactions)	0.743		
Item 16 (Gut feelings)	0.716		
Item 5 (Human nature)	0.691		
Item 8 (Deep feelings)	0.549		
Item 15 (Swiftly)		0.817	
Item 20 (Come to mind)		0.777	
Item 6 (Quickly)		0.731	
Item 9 (No mulling)		0.726	
Item 12 (Experience)		0.702	
Item 3 (Quick)		0.677	
Item 18 (Aware)		0.581	
Item 10 (Detailed plans)			0.789
Item 17 (Action plan)			0.782
Item 1 (Plans important)			0.762
Item 13 (Step-by-step)			0.717
Item 7 (Goals)			0.698
Item 4 (Reasons)			0.496
Item 19 (Hunch)			

Note. Applied rotation method is promax.

RC1: “Feelings-based decision making“, “Intuition“

RC2: “Fast decision making“

RC3: “Planning-based decision making“, “Deliberation“



Factor scores

Matrix with factor scores

Matrix with factor score weights

Matrix with standardized values of variables

Inverse of correlation matrix

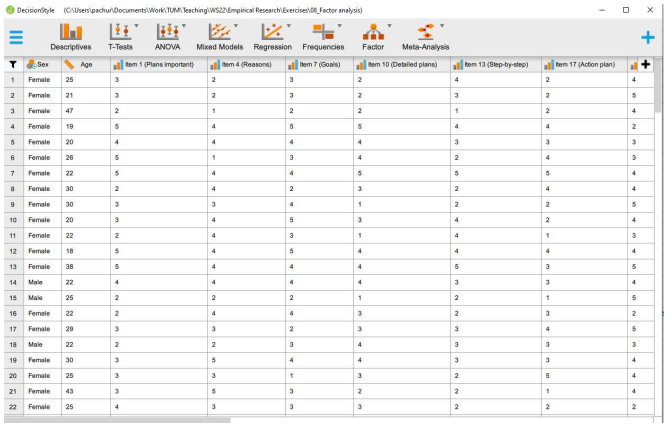
Factor loading matrix **A**

$$\boldsymbol{F} = \boldsymbol{B} \cdot \boldsymbol{Z}$$

$$\boldsymbol{B} = \boldsymbol{R}^{-1} \cdot \boldsymbol{A}$$

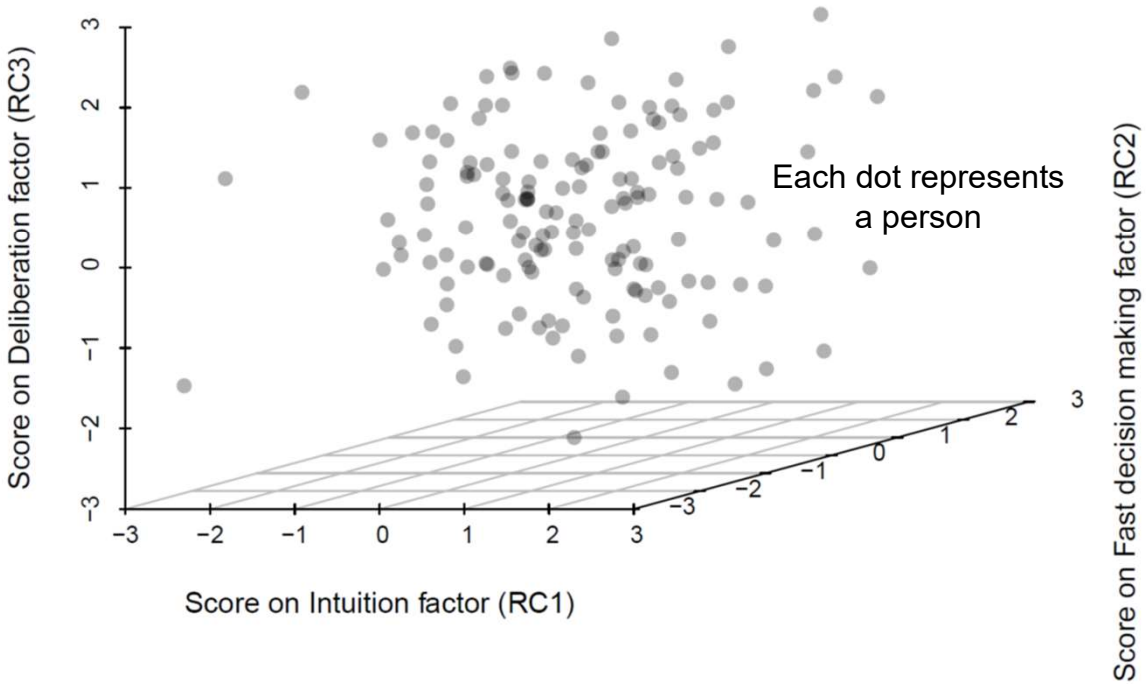
Factor scores

→ Values of the objects on the extracted factors



Originally 21 values for each person

Now 3 values (factor scores) for each person



Preparatory considerations

- The variables should be measured on the interval scale level
- The variables should have roughly normal distributions
- The intercorrelation matrix should contain at least several correlations of .3 or larger (Tabachnick & Fidell, 2016)
- Bartlett‘s test of sphericity

  → Tests the null hypothesis that the variance-covariance matrix is an identity matrix (i.e., in which all correlations between the variables are zero; **test should be significant**, indicating that the variance-covariance matrix is **not** an identity matrix)
- Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy (should be >.6)

  → Quantifies whether there is shared variance among the variables

$$KMO = \frac{\sum\sum_{j \neq k} r_{jk}^2}{\sum\sum_{j \neq k} r_{jk}^2 + \sum\sum_{j \neq k} p_{jk}^2}$$

r: correlation between pairs of variables

p: correlation between pairs of variables with all other variables partialled out

Practical considerations

- Rules of thumb for design
  - At least 10 objects per variable
  - At least 4 variables per factor
- Parameters become stable with ~300 objects (Tabachnick & Fidell, 2007; Comrey & Lee, 1992)
- What matters is the size of the variables‘ communalities for the extracted factor structure: if the communalities are > .6, then even a relatively small number of objects is sufficient (MacCallum et al., 1999)

Self-quiz questions

- What are the goals of factor-analytic techniques?
- What do eigenvalues in a principle component analysis (PCA) represent?
- What is the key idea underlying the scree test? What is the purpose of parallel analysis?
- What are factor loadings? What is meant by uniqueness and how is it related to communality?
- Why is it usually useful to conduct a rotation of the extracted factor solution?
- What is the difference between orthogonal and oblique rotation?
- What are factor scores?
- What is measured with the Bartlett‘s test and the KMO test and what results of these tests are desirable?
- Give two rules of thumb when planning the sample size for a PCA

Background reading for next lecture

Backhaus, K., Erichson, B., Gensler, S., Weiber, R., & Weiber, T. (2021). Cluster analysis. In: K. Backhaus, B. Erichson, S. Gensler, R. Weiber, & T. Weiber, *Multivariate analysis: An application-oriented introduction* (p. 451–530). Springer.