Operations Research and Decision Analysis

Prof. Dr. Rainer Kolisch
Chair of Operations Management
TUM School of Management
Technical University of Munich

General Information

Audience:
- Bachelor in Management andTechnology (MGT001374)
- Master in Management (MGT001464)
- Students from other programs of TUM
- Exchange students

Parts of the module:
- Lecture: Thursday Audimax (Prof. Dr. Rainer Kolisch)
- Exercise: Friday Audimax (Baturhan Bayraktar, Andreas Hagn, Deepthi Padinjaroot)
- Tutorial: Different dates (Student assistants); Information on how to enroll is given in the video

Learning material:
- Lecture slides
- Manuscript
- Exercise questions
- Videos of the lectures and the exercises
- Additional references given in the manuscript and provided through moodle

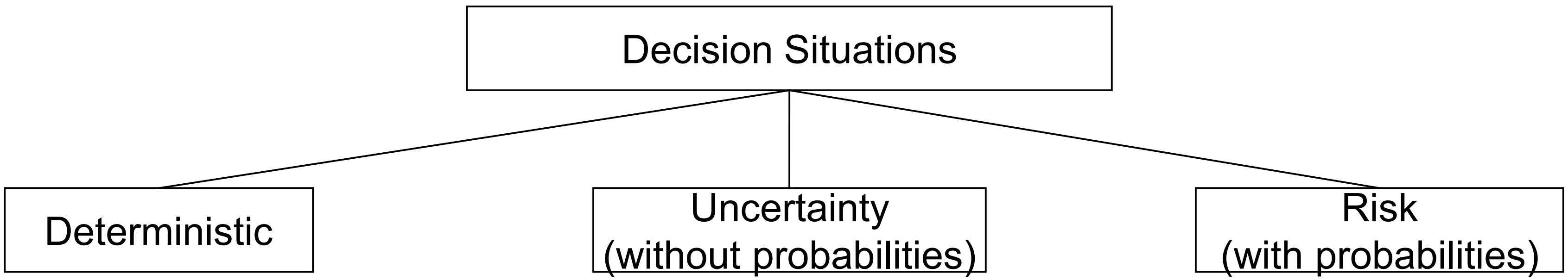
Grading:
- 90 minute written exam in physical presence. Date: February 27, 2026 at 2 pm.

Further information on the organization of the course:
- Video in moodle and in the first exercise on Friday, October 24.

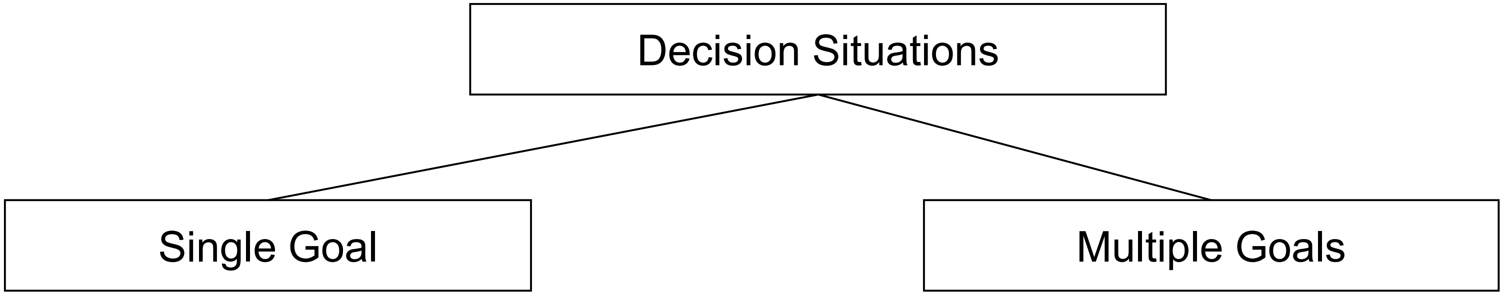
1 Decision Analysis

1.1 Decision Situations

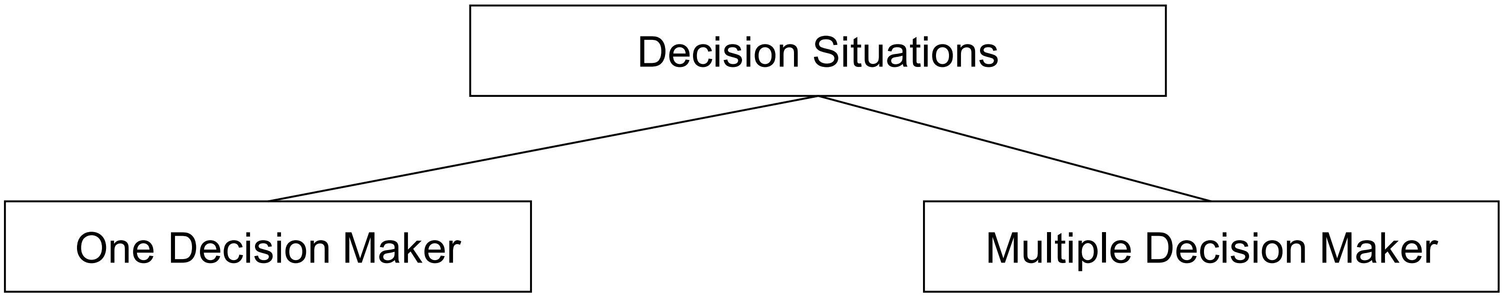
Level of Uncertainty



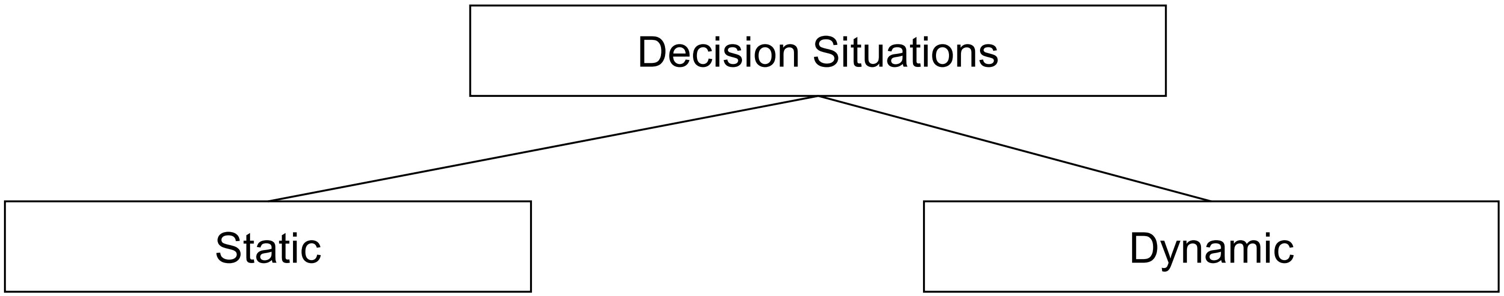
Number of Goals



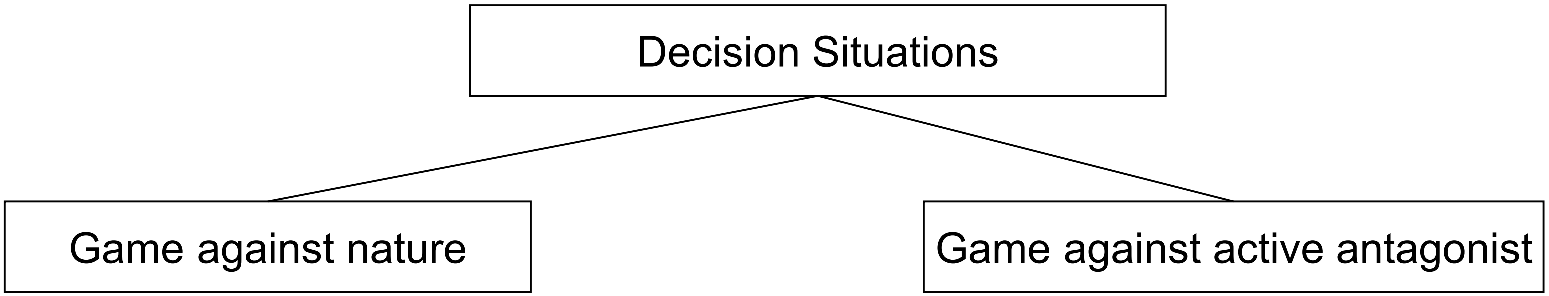
Number of Decision Makers



Development over Time



Type of Antagonist



1.2 Decision Making under Uncertainty

Characterization of the Decision Situation

- Uncertainty (multiple scenarios without probabilities)
- Single goal
- Single decision maker
- Static decision context
- Game against nature

Decision Matrix: Actions

a_1

…

a_i

…

a_m

m alternatives. One of them has to be chosen.

Decision Matrix: Scenarios

	s_1	…	s_j	…	s_n
a_1					
…					
a_i					
…					
a_m					

n scenarios. One will occur.

Decision Matrix: Outcomes

	s_1	…	s_j	…	s_n
a_1	$e_{1,1}$	…	$e_{1,j}$	…	$e_{1,n}$
…					
a_i	$e_{i,1}$	…	$e_{i,j}$	…	$e_{i,n}$
…					
a_m	$e_{m,1}$	…	$e_{m,j}$	…	$e_{m,n}$

Sequence of actions and events: Action a_i → Scenario s_j → Outcome $e_{i,j}$

Example: Newsvendor Problem

Assumptions:

- Buying one copy for 20 Cent
- Selling one copy for 25 Cent
- Demand is between 6 and 10
- Not sold copies don't have value

Problem:

How many copies should be bought in order to maximize profit?

Development of the Decision Matrix

Full Decision Matrix for the Newsvendor Problem

Actions: Number of copies bought (a_i); Scenarios (s_j); Demand

a_i \ s_j (Scenarios)	1	2	3	4	5
Demand	6	7	8	9	10
1	5	5	5	5	5
2	10	10	10	10	10
3	15	15	15	15	15
4	20	20	20	20	20
5	25	25	25	25	25
6	30	30	30	30	30
7	10	35	35	35	35
8	-10	15	40	40	40
9	-30	-5	20	45	45
10	-50	-25	0	25	50
11	-70	-45	-20	5	30
…	…	…	…	…	…

Reducing the Decision Matrix to Efficient Actions

Demand: 6, 7, 8, 9, 10

Scenarios s_j (Copies sold): 1, 2, 3, 4, 5

Actions a_i: Number of copies bought

a_i \ s_j	1	2	3	4	5
1	5	5	5	5	5
2	10	10	10	10	10
3	15	15	15	15	15
4	20	20	20	20	20
5	25	25	25	25	25
6	30	30	30	30	30
7	10	35	35	35	35
8	-10	15	40	40	40
9	-30	-5	20	45	45
10	-50	-25	0	25	50
11	-70	-45	-20	5	30
…	…	…	…	…	…

Options 6 through 10 must be considered

Efficient and Dominated Actions

Reduction of the Decision Matrix to Efficient Actions

	6	7	8	9	10
			s_j		
a_i	1	2	3	4	5
1	5	5	5	5	5
2	10	10	10	10	10
3	15	15	15	15	15
4	20	20	20	20	20
5	25	25	25	25	25
6	30	30	30	30	30
7	10	35	35	35	35
8	-10	15	40	40	40
9	-30	-5	20	45	45
10	-50	-25	0	25	50
11	-70	-45	-20	5	30
…	…	…	…	…	…

- Delete dominated actions
- Renumber efficient actions

			s_j		
a_i	1	2	3	4	5
1	30	30	30	30	30
2	10	35	35	35	35
3	-10	15	40	40	40
4	-30	-5	20	45	45
5	-50	-25	0	25	50

Which action would you choose?

Decision Rules

- Maximin
- Maximax
- Hurwicz
- Minimax Regret
- Laplace

Note:

- There is no optimal rule.
- The choice of the rule reflects the decision makers attitude towards risk.

Maximin-Rule

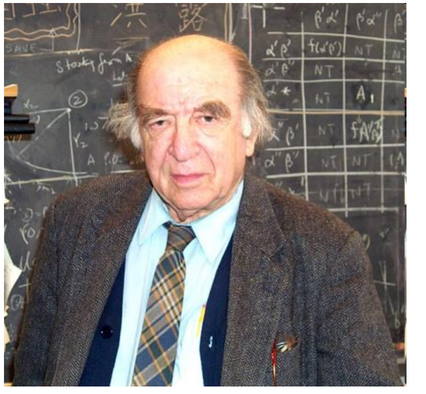
a_i \ s_j	1	2	3	4	5
1	30	30	30	30	30
2	10	35	35	35	35
3	-10	15	40	40	40
4	-30	-5	20	45	45
5	-50	-25	0	25	50

Maximax-Rule

		s_j				
		1	2	3	4	5
	1	30	30	30	30	30
	2	10	35	35	35	35
a_i	3	-10	15	40	40	40
	4	-30	-5	20	45	45
	5	-50	-25	0	25	50

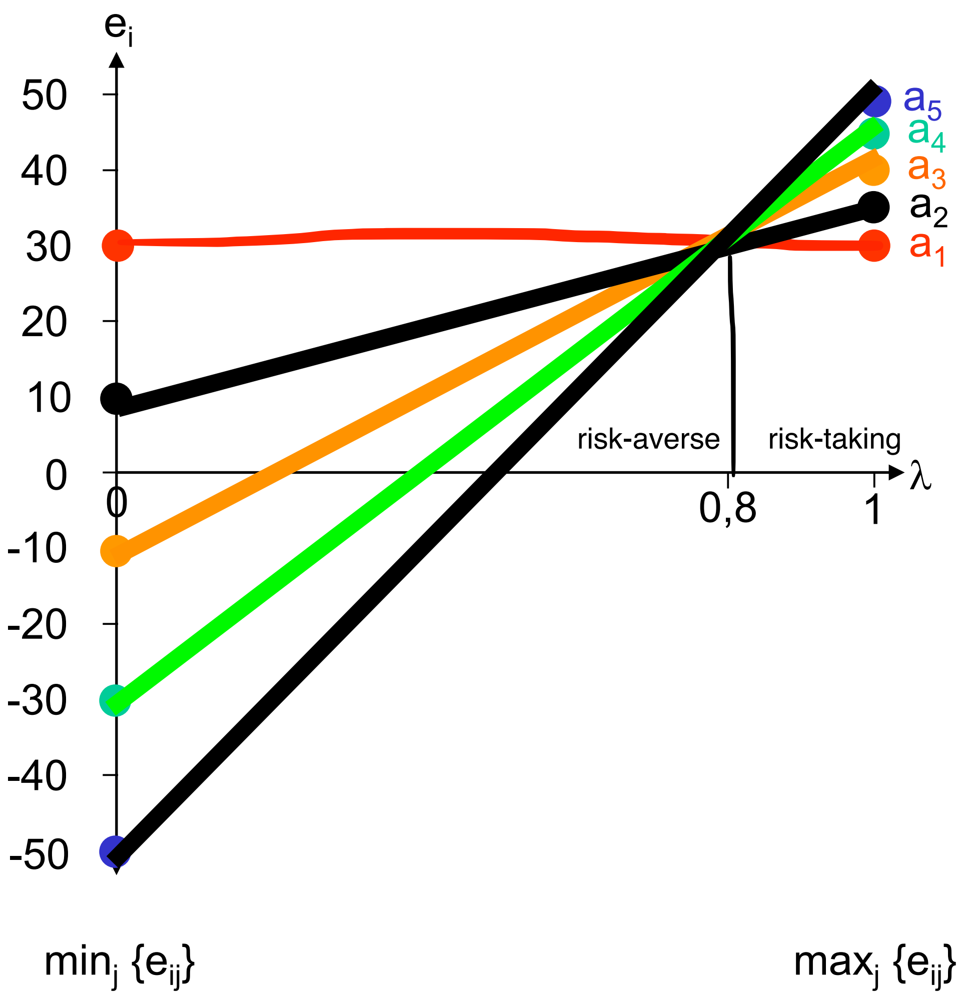
Hurwicz-Rule

		s_j 1	2	3	4	5	$\max_j \{e_{i,j}\}$	$\min_j \{e_{ij}\}$	e_i
	1	30	30	30	30	30			
	2	10	35	35	35	35			
a_i	3	-10	15	40	40	40			
	4	-30	-5	20	45	45			
	5	-50	-25	0	25	50			


Leonid Hurwicz (1917-2008)
University of Minnesota,
Laureate of the Nobel Price

For each action linear combination of max- and min-value: $e_i = \lambda \cdot \max_j \{e_{ij}\} + (1-\lambda) \cdot \min_j \{e_{ij}\}$

Hurwicz: Value of Actions Depending on λ



Minimax Regret-Rule: Regret

2) Scenario s_4 occurs

1) DM chooses action a_2

a_i \ s_j	1	2	3	4	5
1	30	30	30	30	30
2	10	35	35	35	35
3	-10	15	40	40	40
4	-30	-5	20	45	45
5	-50	-25	0	25	50

3) Result

Best possible result for the scenario which occurred

4) Regret = Difference in result for chosen action vs. ex post optimal action.

$$r_{2,4} = 45 - 35 = 10$$

Minimax Regret Rule: Deriving the Regret Matrix

Decision Matrix (e)

		s_j				
		1	2	3	4	5
a_i	1	30	30	30	30	30
	2	10	35	35	35	35
	3	-10	15	40	40	40
	4	-30	-5	20	45	45
	5	-50	-25	0	25	50

Regret Matrix (r)

		s_j				
		1	2	3	4	5
a_i	1	0				
	2	20				
	3	…				
	4					
	5					

Minimax Regret-Rule

Regret Matrix (r)

a_i \ s_j	1	2	3	4	5
1	0	5	10	15	20
2	20	0	5	10	15
3	40	20	0	5	10
4	60	40	20	0	5
5	80	60	40	20	0

Laplace-Rule

		s_j				
		1	2	3	4	5
	1	30	30	30	30	30
	2	10	35	35	35	35
a_i	3	-10	15	40	40	40
	4	-30	-5	20	45	45
	5	-50	-25	0	25	50

Pierre-Simon Laplace (1749-1827)
(French Mathematician)

1.3 Decision Making under Risk: Expected Utility Theory

Characterization of the Decision Situation

- Risk: Multiple scenarios with probabilities
- Single goal
- Single decision maker
- Static decision context
- Game against nature

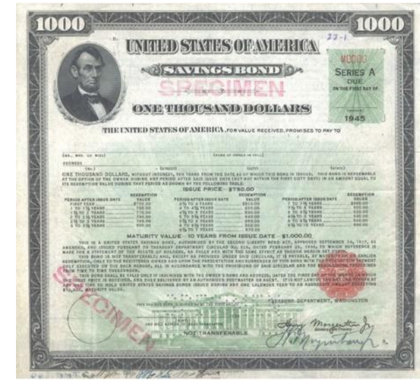
Extended Decision Matrix

Scenarios

Actions		s_1	…	s_j	…	s_n
	a_1	$e_{1,1}$	…	$e_{1,j}$	…	$e_{1,n}$
	…					
	a_i	$e_{i,1}$	…	$e_{i,j}$	…	$e_{i,n}$
	…					
	a_m	$e_{m,1}$	…	$e_{m,j}$	…	$e_{m,n}$

Example Extended Decision Matrix: Share vs. Bond

Payoff:
- 30K Euro (50% probability, scenario: „Top“)
- 0K Euro (50% probability, scenario: „Flop“).



Payoff: 10K

	s_1: Top p_1=	s_2: Flop p_2=
a_1: Share		
a_2: Bond		

Representation of an Action as Lottery

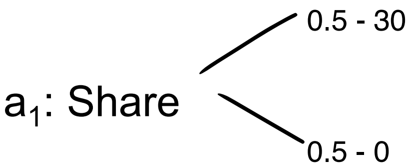
Decision Matrix

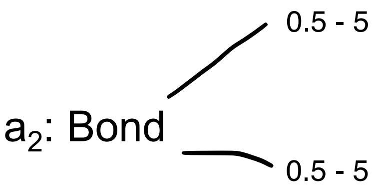
Lottery

	s_1	…	s_j	…	s_n
	p_1	…	p_j	…	p_n
a_1	$e_{1,1}$	…	$e_{1,j}$	…	$e_{1,n}$
…					
a_i	$e_{i,1}$	…	$e_{i,j}$	…	$e_{i,n}$
…					
a_m	$e_{m,1}$	…	$e_{m,j}$	…	$e_{m,n}$

Example Lottery: Share vs. Bond

	s_1: Top p_1= 0.5	s_2: Flop p_2= 0.5
a_1: Share	30K	0
a_2: Bond	10K	10K





Which lottery would you choose?

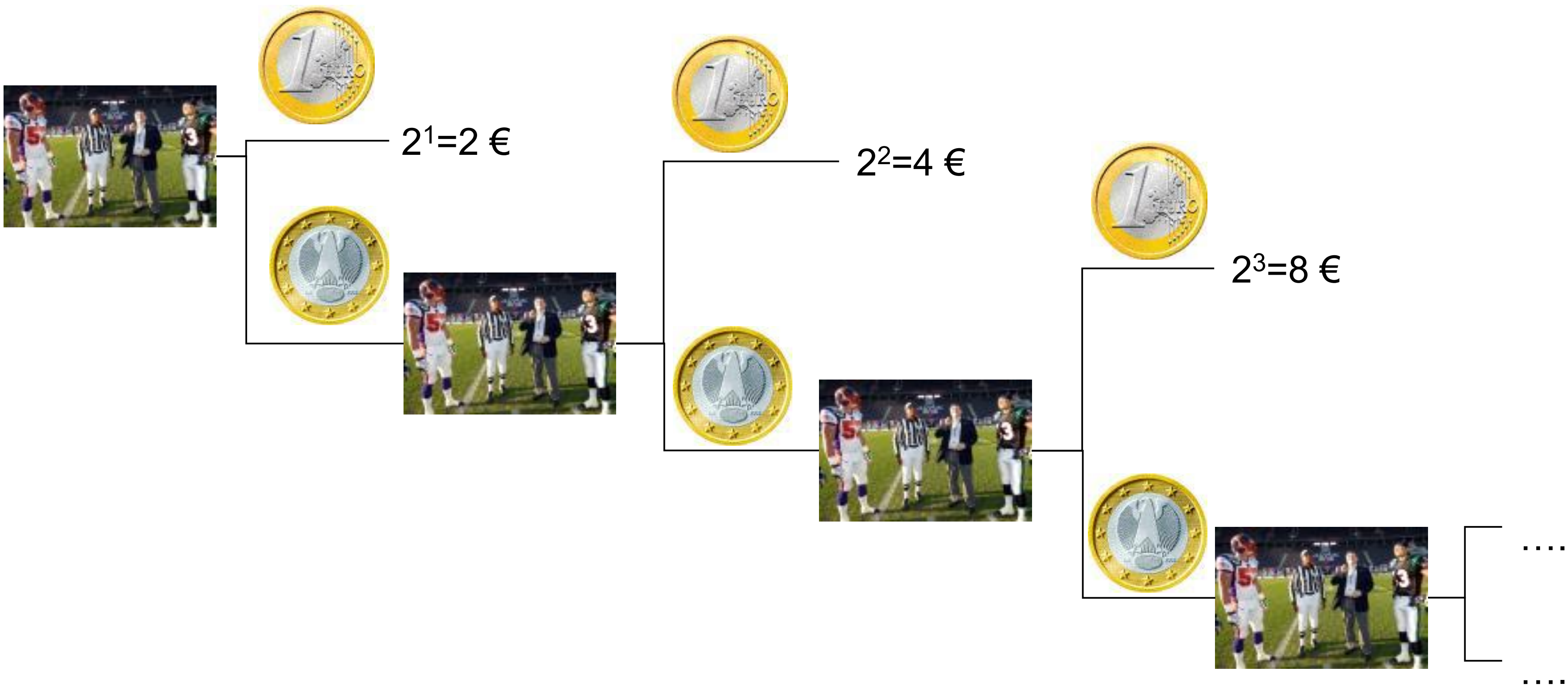
Valuing Lotteries

Daniel Bernoulli (1700-1782)
(Swiss Mathematician)

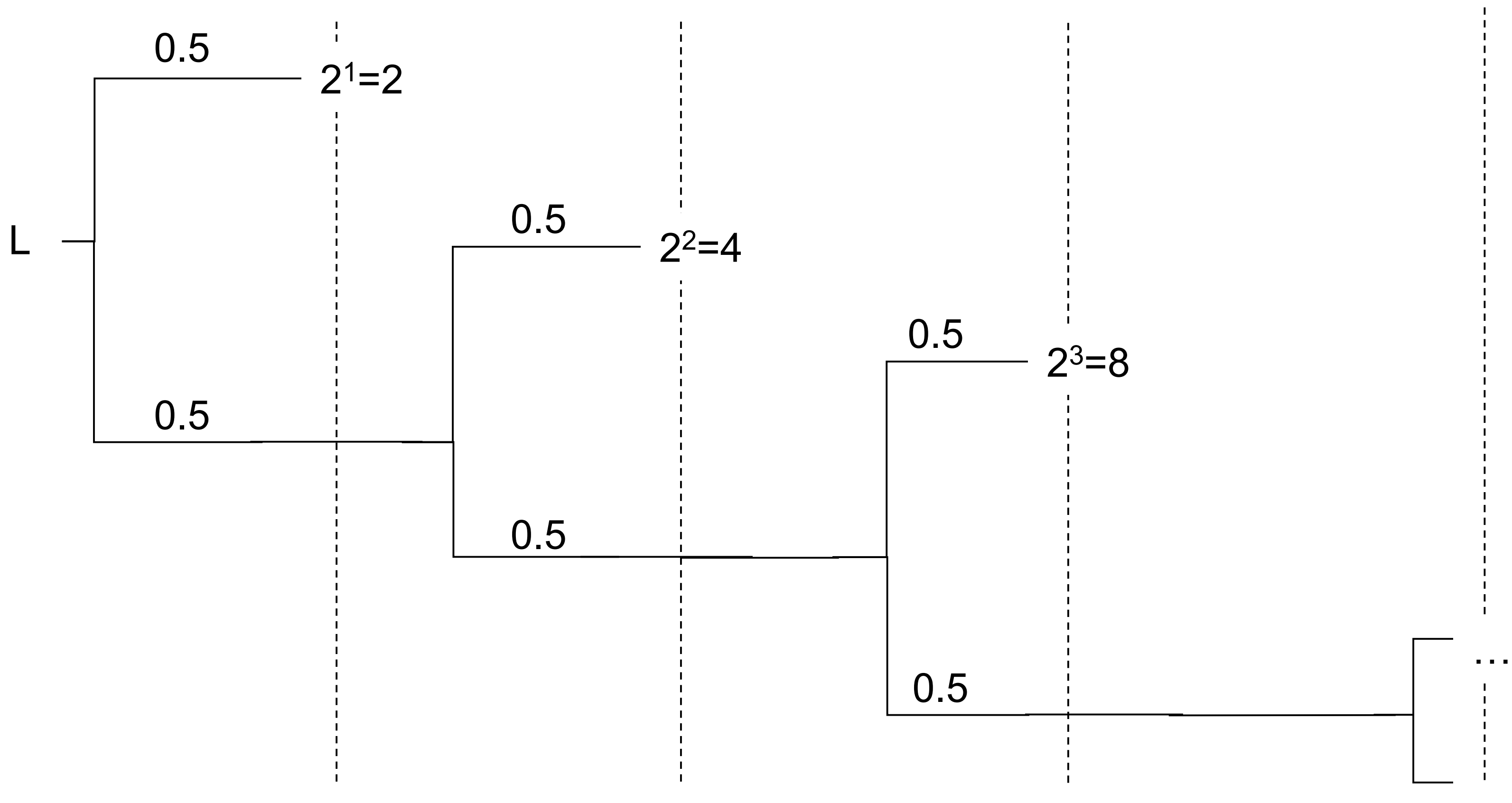
St. Petersburg Game

Rules of the game:

- Tossing a fair coin
- In case of number, 2 € will be paid and the game is over.
- In case of eagle, the coin will be tossed again.
- The payoff at the end of the game is $2^{(\#tosses)}$ €.



Expected Value of the St. Petersburg Lottery

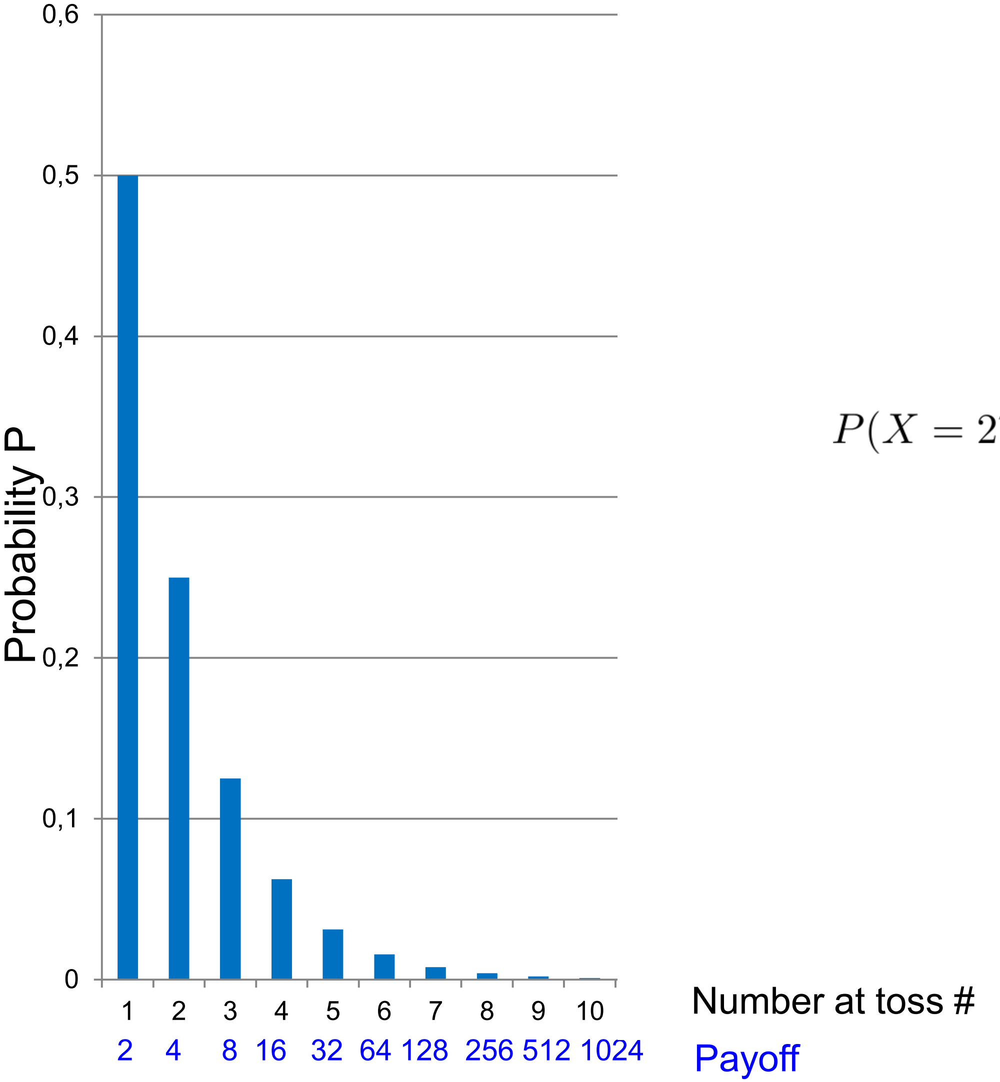


Number at toss # 1 2 3

Payoff ….

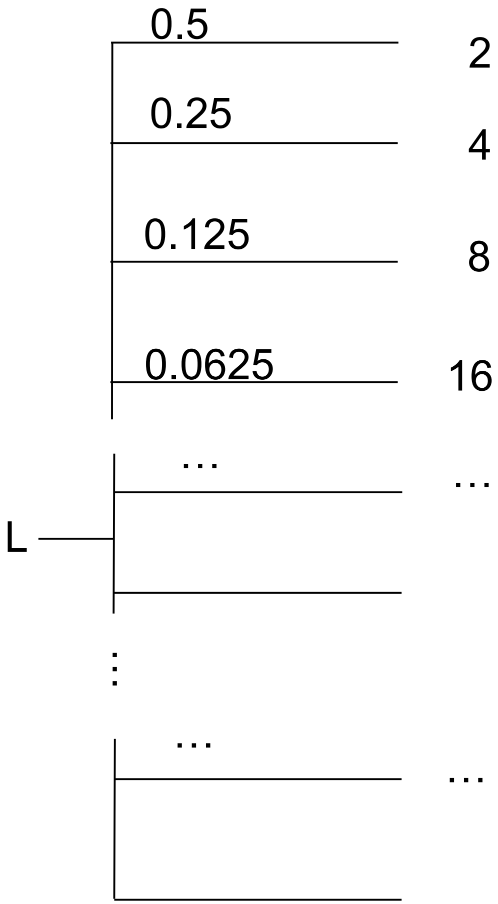
Probability ….

Probability Distribution

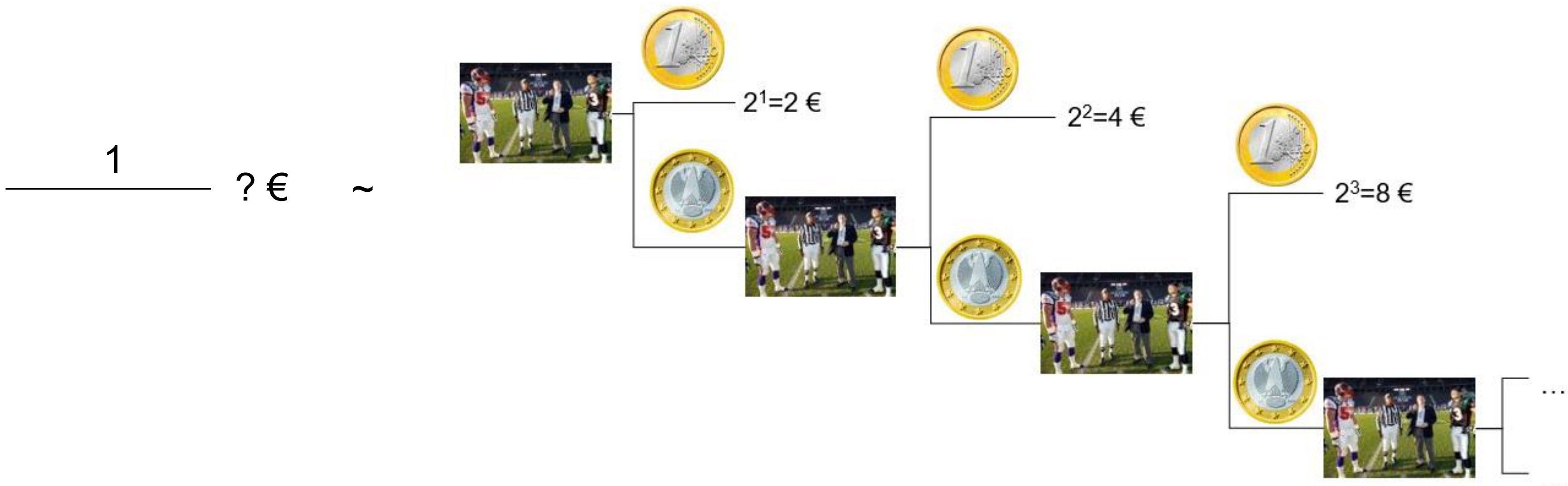


$$P(X = 2^i) = 0.5^i \qquad i = 1, \ldots, \infty$$

Expected Value of the St. Petersburg Lottery

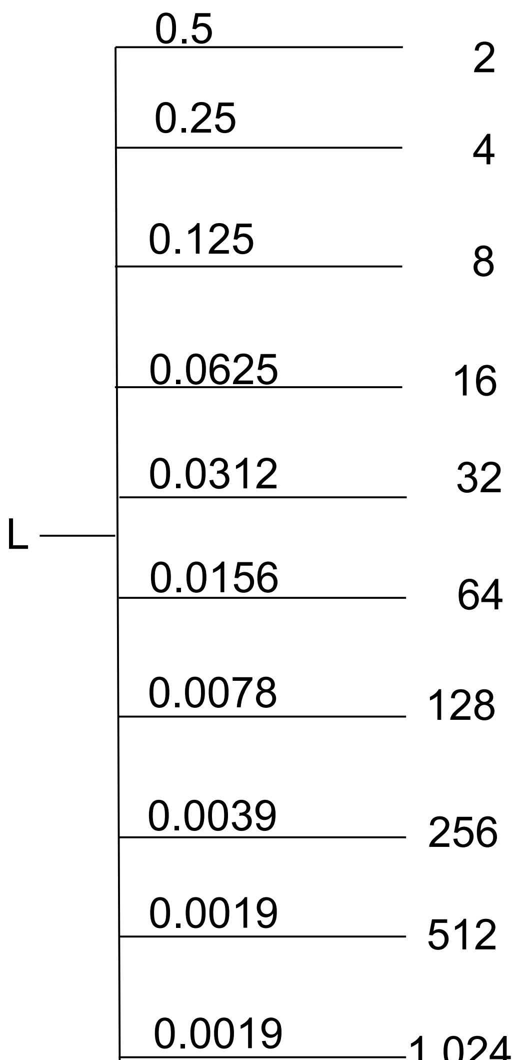


Your Valuation of the St. Petersburg Game



For which certainty payoff are you indifferent to the St. Petersburg lottery ?

Exp. Value St. Petersburg Lottery with max 10 Tosses

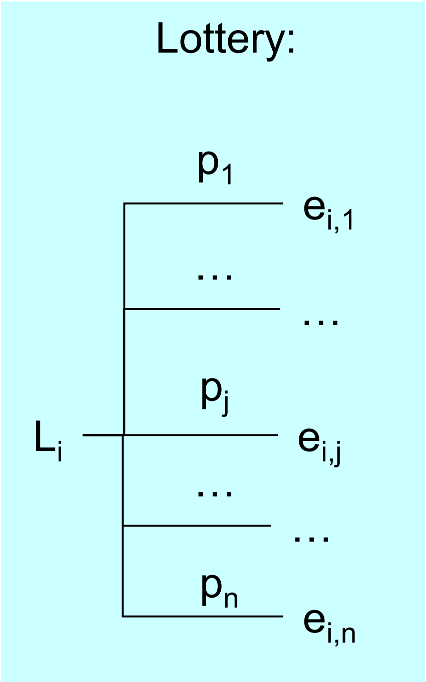


Expected Utility

- Developed by Bernoulli in 1738
- Axioms derived by Von Neumann and Morgenstern in 1947

Expected value of lottery L_i:

Expected utility of lottery L_i :



► For the expected utility we need a utility function u(.), which gives for each risky result e an utility u(e).

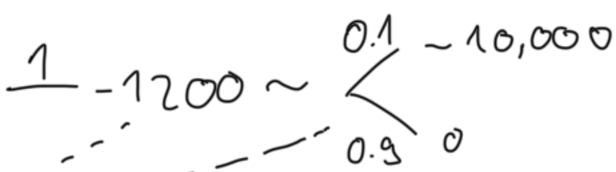
Certainty Equivalent

The certainty equivalent CE(L) of a lottery L is the number CE(L) such that the DM is indifferent between the lottery L and receiving a certain payment of CE(L).

Risk Premium

The risk premium RP(L) of a lottery L is the difference between the expected value of the lottery EV(L) and the certainty equivalent of the lottery CE(L).

EX: Insurance



$EV(L) = 0.1 \cdot -10{,}000 + 0.9 \cdot 0 = -1000$

$CE(L) = -1200$

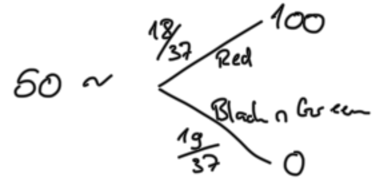
$RP(L) = -1000 - (-1200) = 200$

CE defined by actor, EV by circumstance

→ Over many people, risk averages out

→ Risk neutral

EX: Roulette



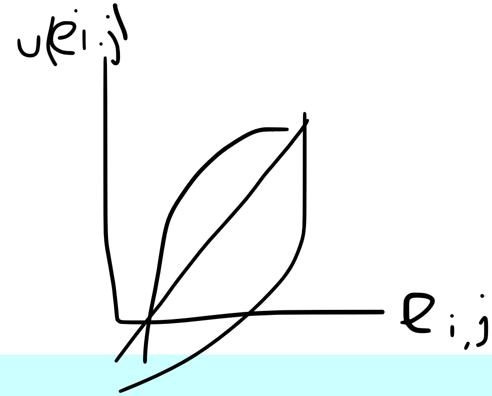
$EV(L) = \frac{18}{37} \cdot 100 = 48.6$

$CE(L) = 50$

$RP(L) = 48.6 - 50 = -1.4$

→ Risk-seeking

Utility Function u(e)



Best outcome of the decision matrix:

Worst outcome of the decision matrix:

1. Scaling to [0,1]:

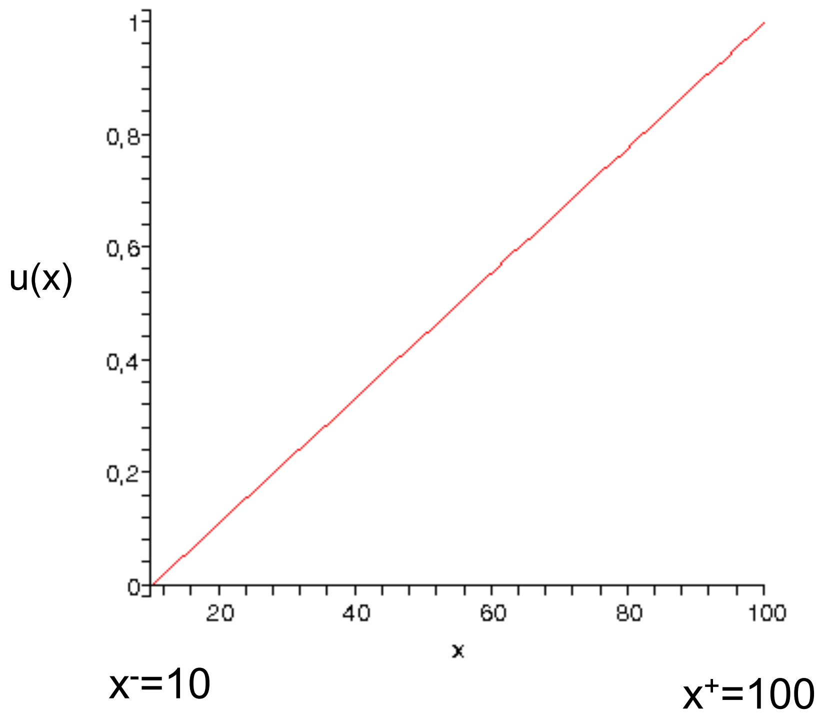
2. Ordering of alternatives:

Decision matrix:

		s_1	…	s_j	…	s_n
		p_1	…	p_j	…	p_n
a_1		$e_{1,1}$	…	$e_{1,j}$	…	$e_{1,n}$
…						
a_i		$e_{i,1}$	…	$e_{i,j}$	…	$e_{i,n}$
…						
a_m		$e_{m,1}$	…	$e_{m,j}$	…	$e_{m,n}$

Example of a Linear Utility Function

$$u(x) = -\frac{1}{9} + \frac{1}{90}x$$

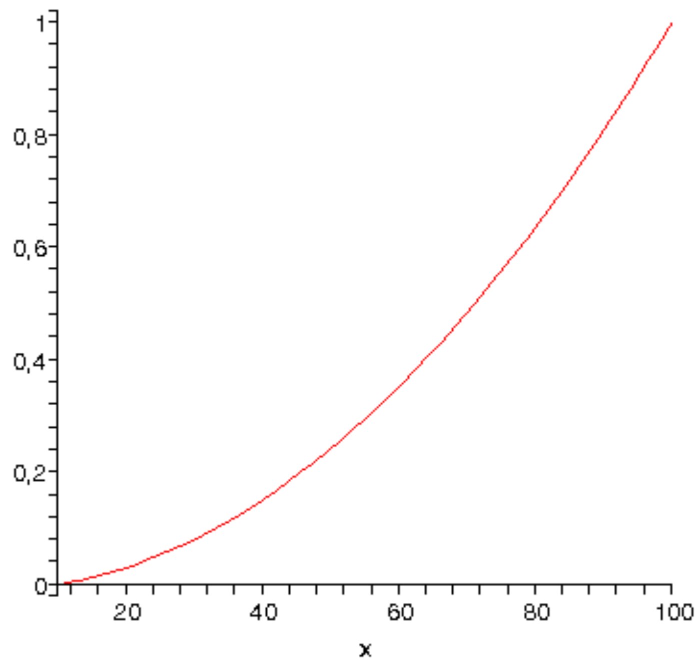


► Increase of utility (gradient of the function) is positive and constant.

Example of a Convex Utility Function

$$u(x) = -\frac{1}{99} + \frac{1}{9900} x^2$$

u(x)



$x^- = 10$

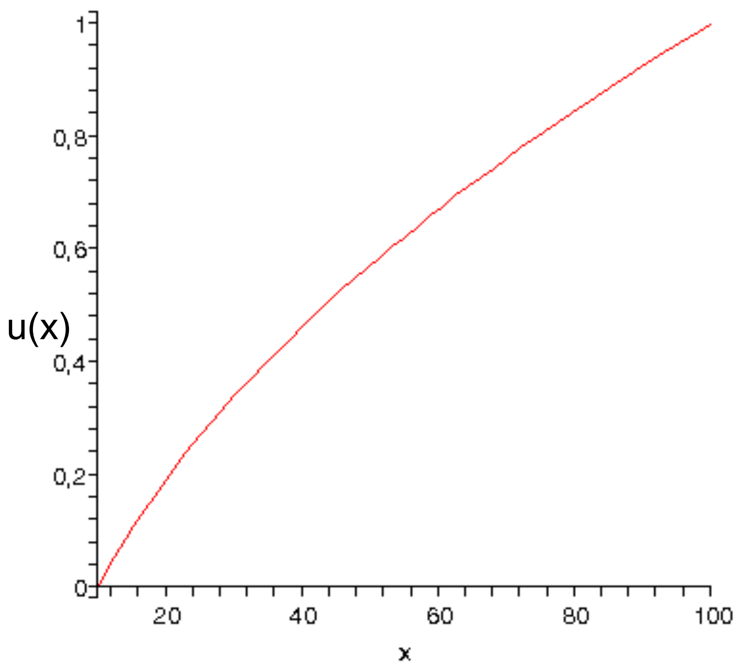
$x^+ = 100$

► Increase of utility is positive and increasing with x

Recap: Convex Function

Example of a Concave Utility Function

$u(x) = -0.46 + 0.146 \cdot x^{0,5}$



$x^- = 10$ $x^+ = 100$

- ► Increase of utility is positive and decreasing with x

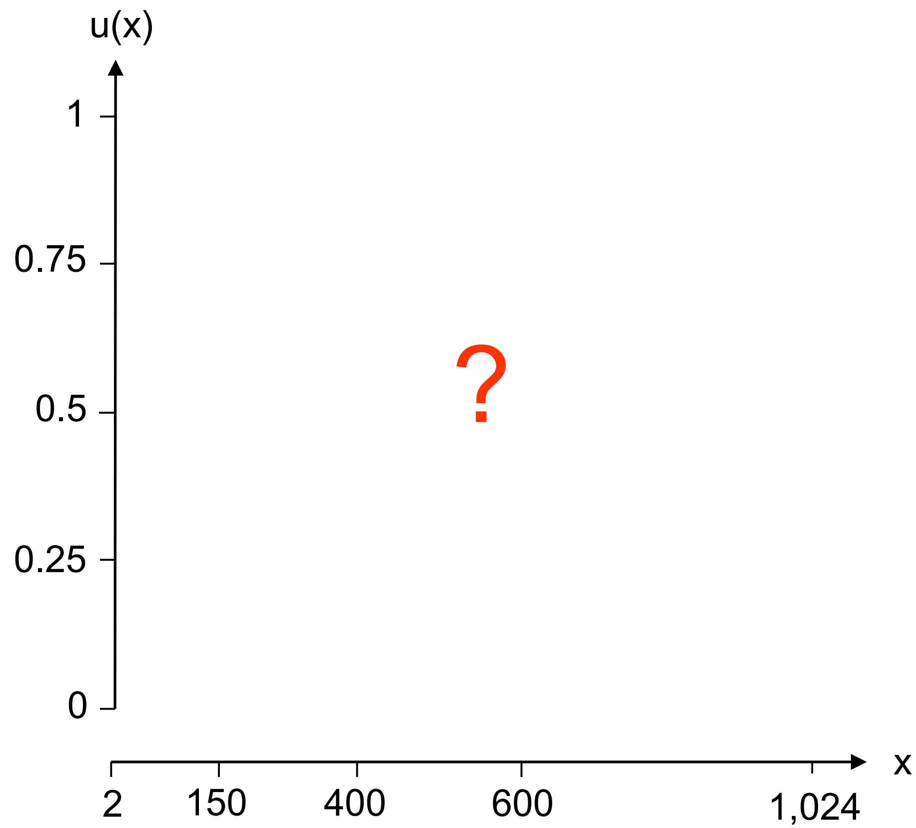
Recap: Concave Function

Constructing the Utility Function u(x) with the Certainty Equivalent Method

- Constructing a utility function for one decision maker and a specific decision problem.
- As specific decision problem, we will use the St. Petersburg game with a maximum number of 10 tosses.
- Central idea: The decision maker is asked for the certainty equivalent of a series of lotteries.

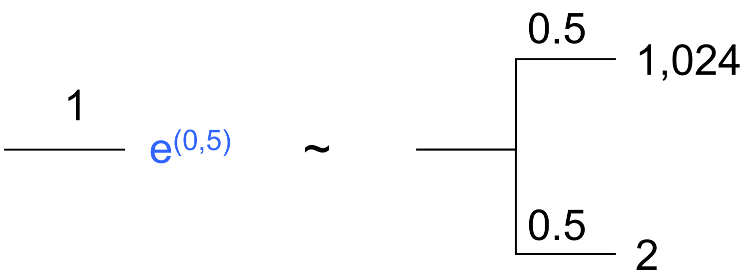
Maximum outcome:

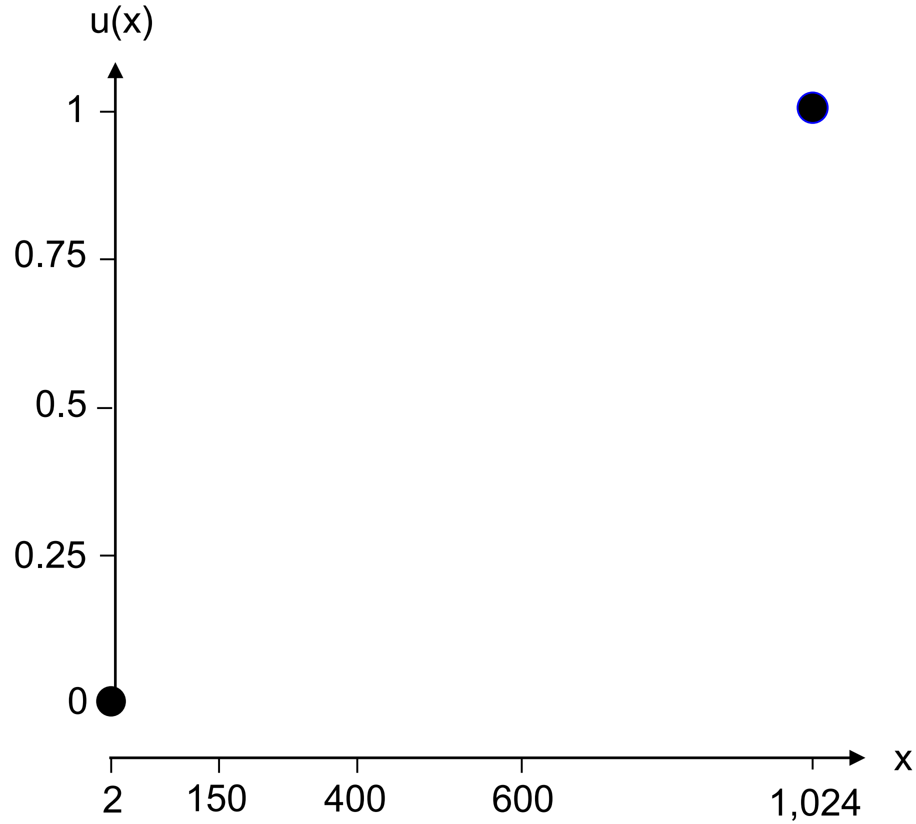
Minimum outcome:



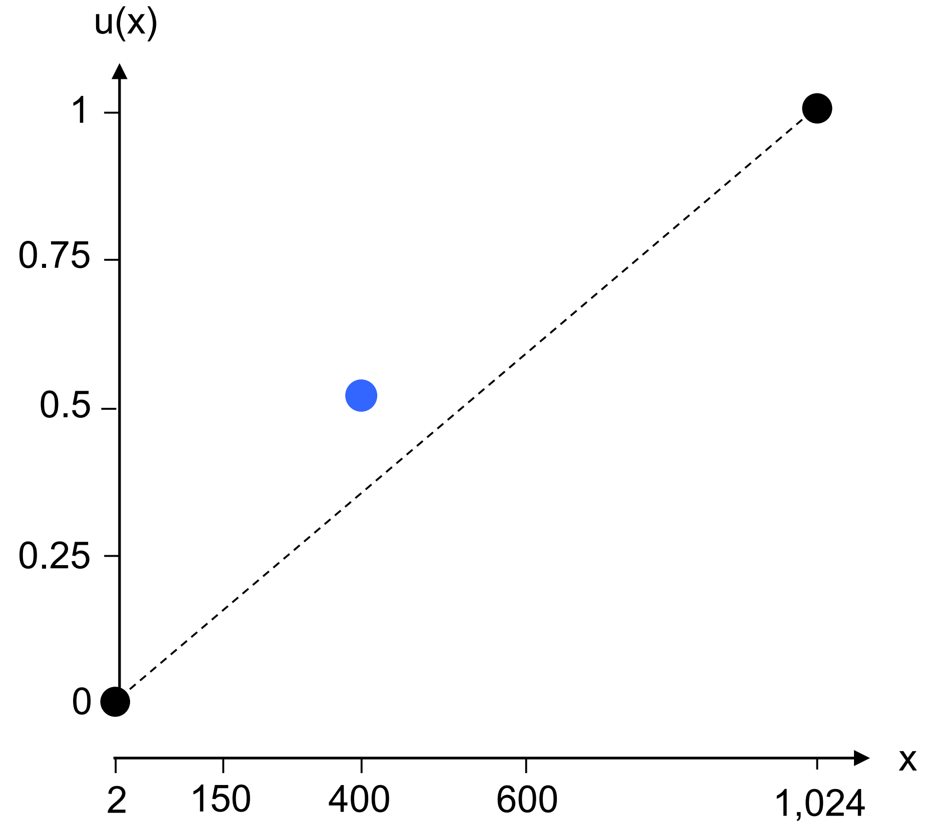
Constructing the Utility Function u(x)

We want to determine the non risky payment $e^{(0,5)}$ for which the decision maker (DM) is indifferent between the two lotteries.

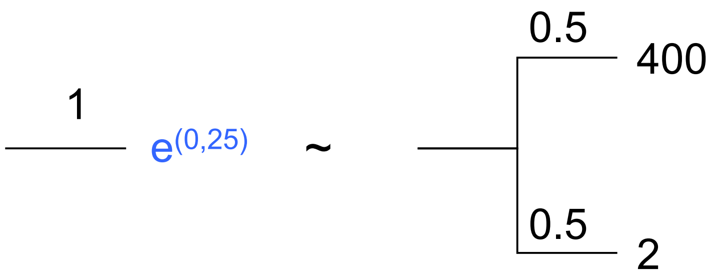


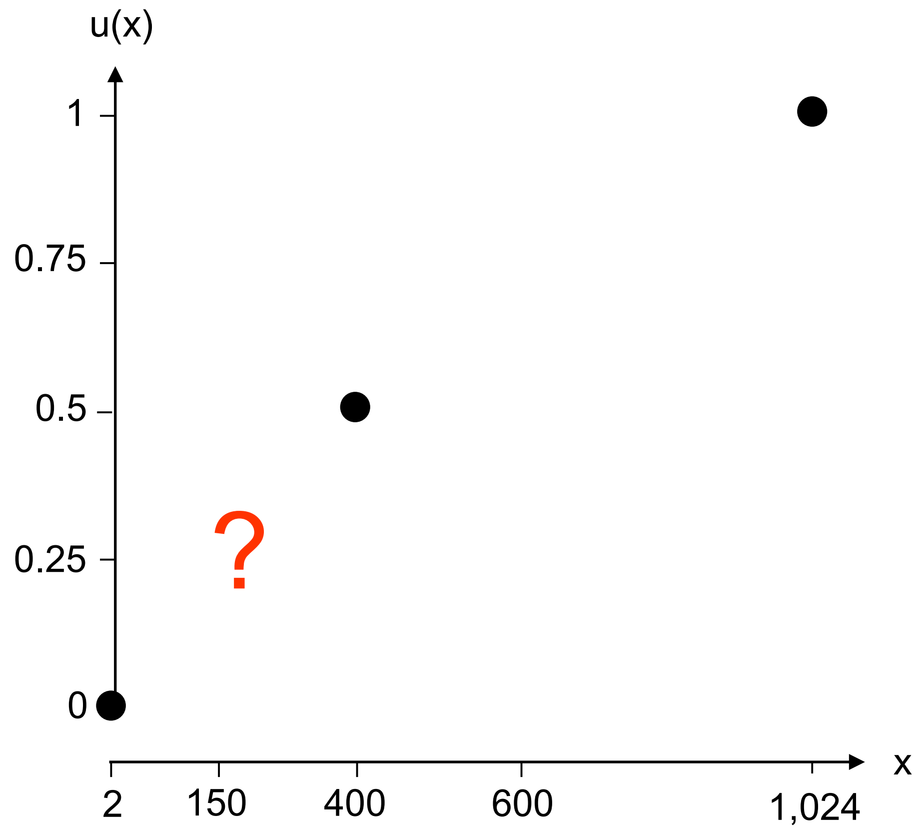


With the First Point of Support We Can Determine the Type of the Utility Function

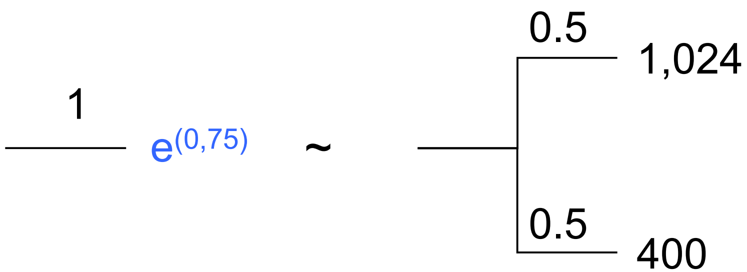


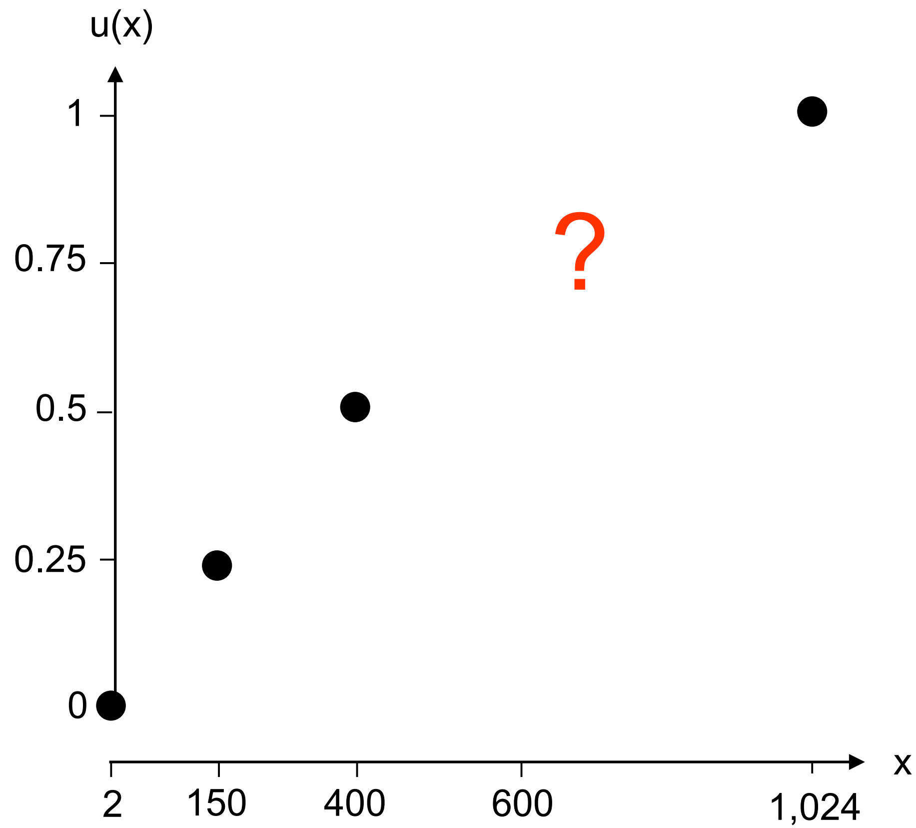
Constructing the Utility Function u(x)



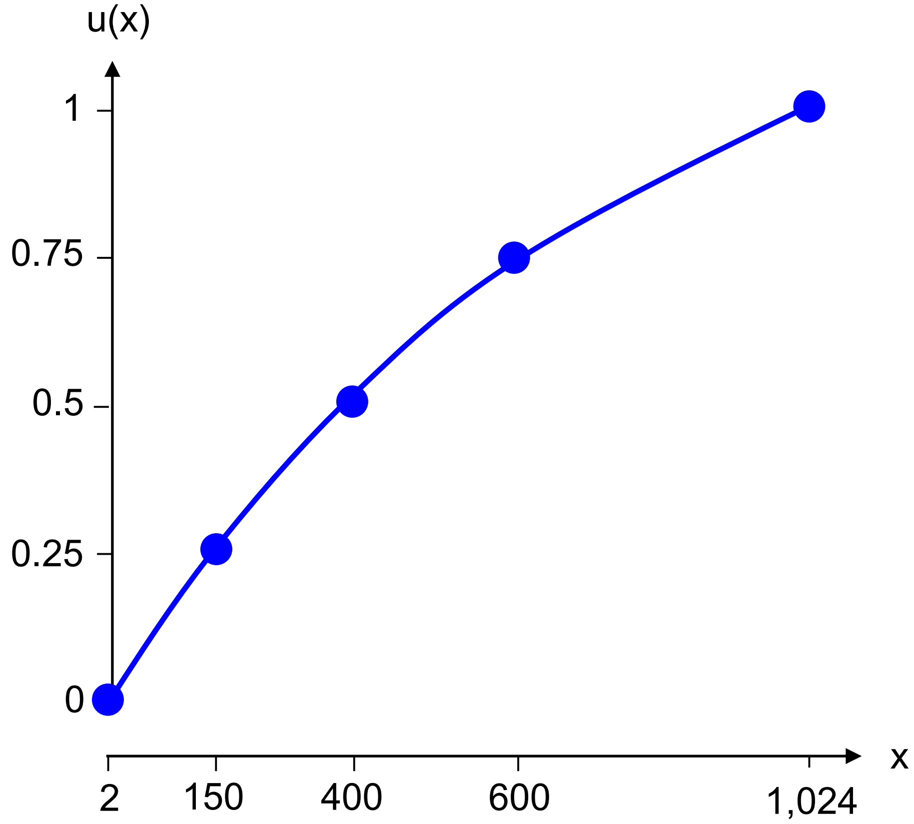


Constructing the Utility Function u(x)





Resulting Utility Function



Arrow-Pratt Measure

The Arrow-Pratt Measure reveals the risk attitude of the DM at x:

Necessary conditions:

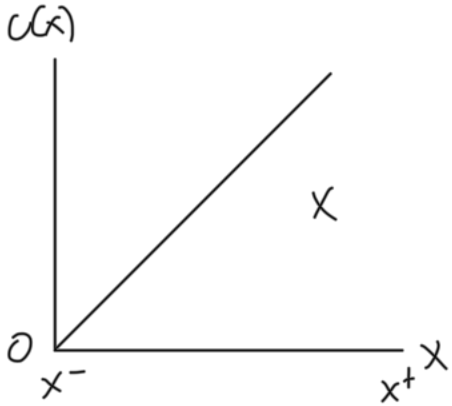
Arrow-Pratt Measure for Different Types of Utility Functions

Recap:

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

Risk neutral

Linear utility function



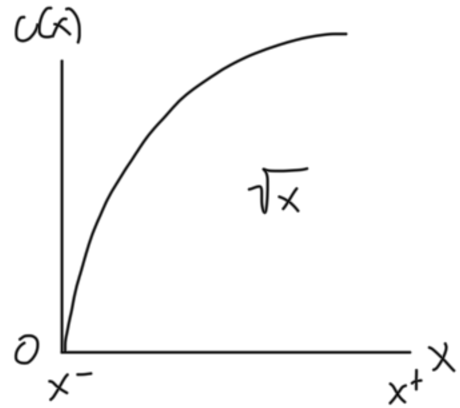
$u'(x) > 0$ for $x^- \leq x \leq x^+$

$u''(x) = 0$

$AP(x) = 0$

Risk-averse

Concave utility function



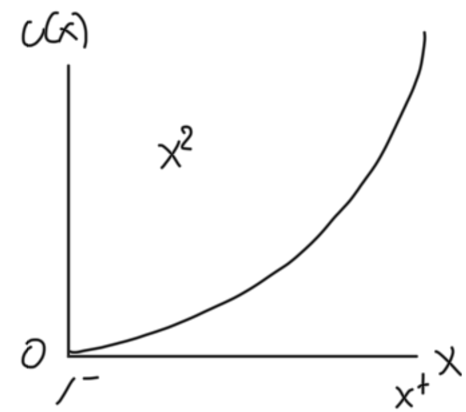
$u'(x) > 0$

$u''(x) < 0$

$AP(x) > 0$

Risk-seeking

Convex utility function



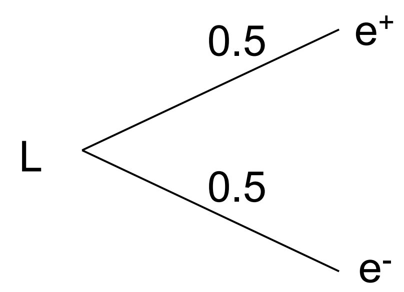
$u'(x) > 0$

$u''(x) > 0$

$AP(x) < 0$

Type of Decision Makers

- Three different type of DMs:
  - Risk avers
  - Risk seeking
  - Risk neutral
- We characterize the three type of DM with lottery

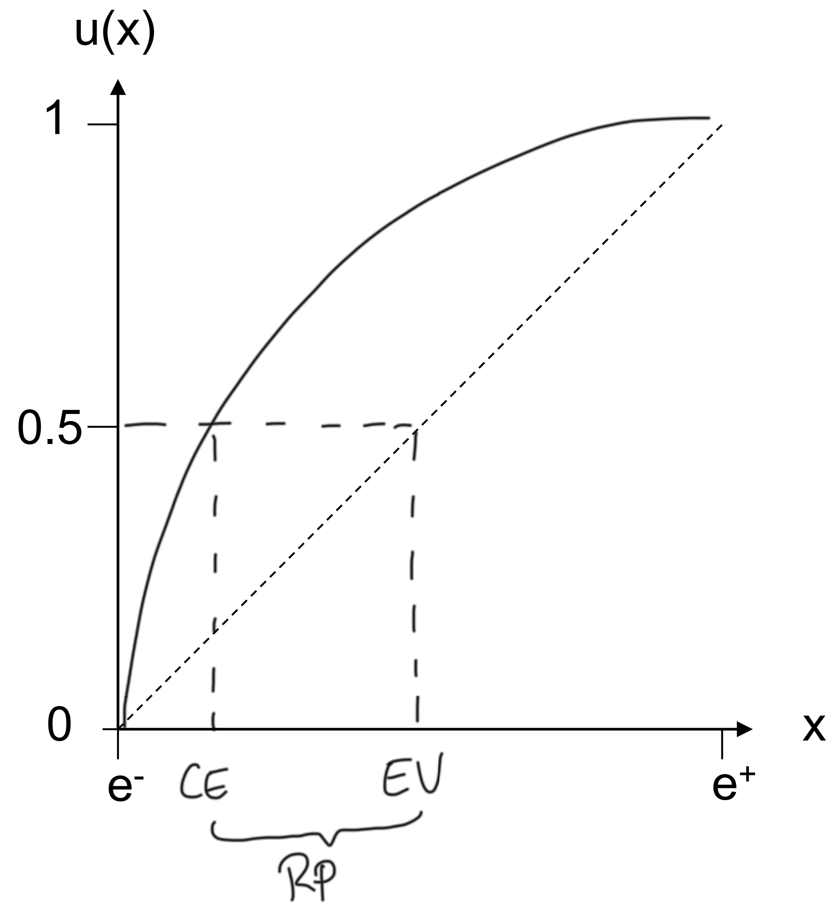


Risk Averse Decision Maker

Convex

RP > 0

AP(x) < 0 for all x

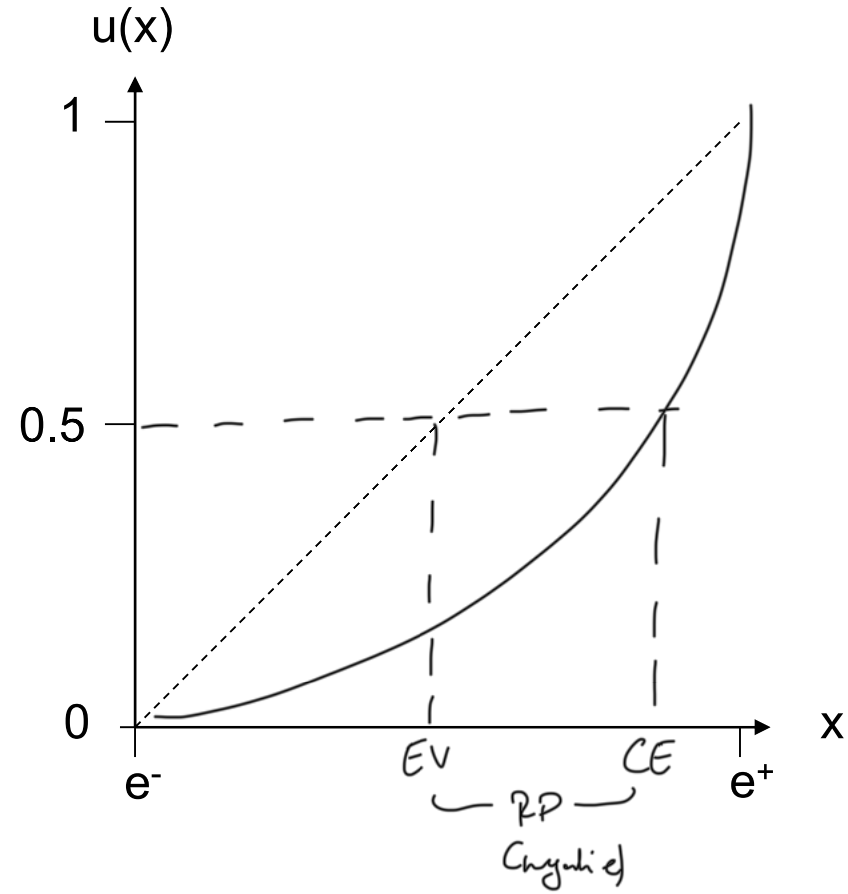


Risk Seeking Decision Maker

Concav

RP<0

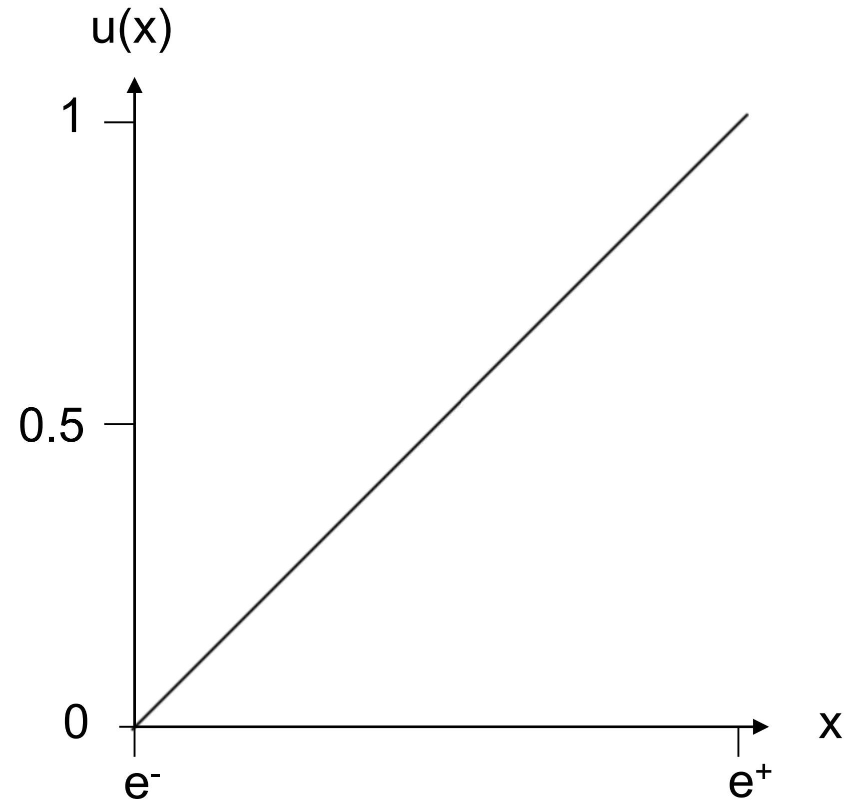
AP(x) > 0 for all x



Risk Neutral Decision Maker

RP = 0

AP(x) = 0



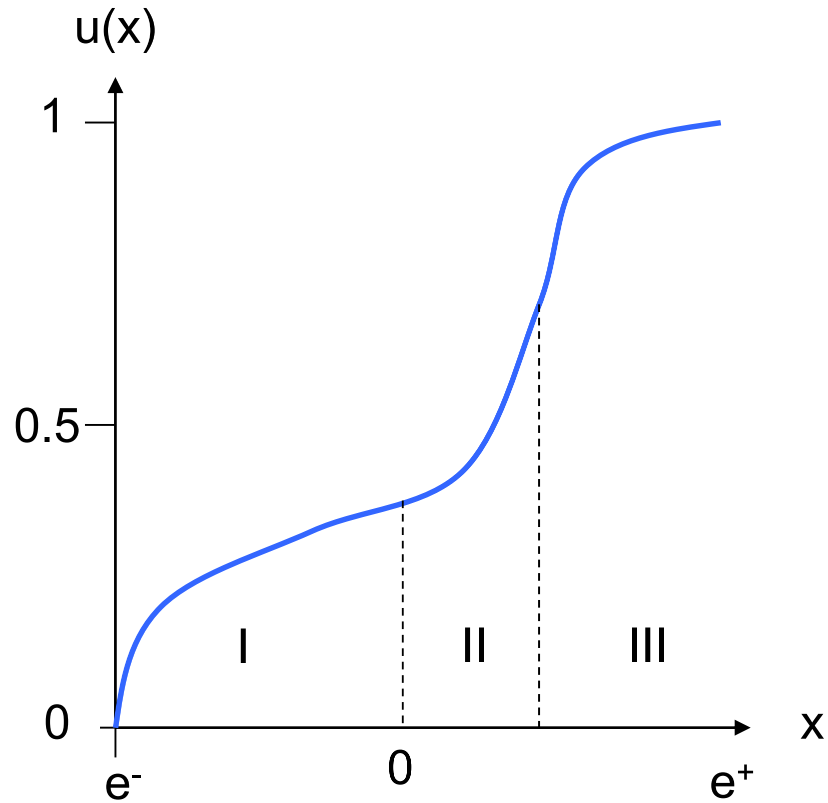
Empirically Observed Utility Functions

Friedman and Savage (1948) observed that DM do not have only one type of utility function. Rather, the utility function is mixed and depends on the specific situation.

Situation I:

Situation II:

Situation III:



Selection of Risky Actions Using Utility Functions

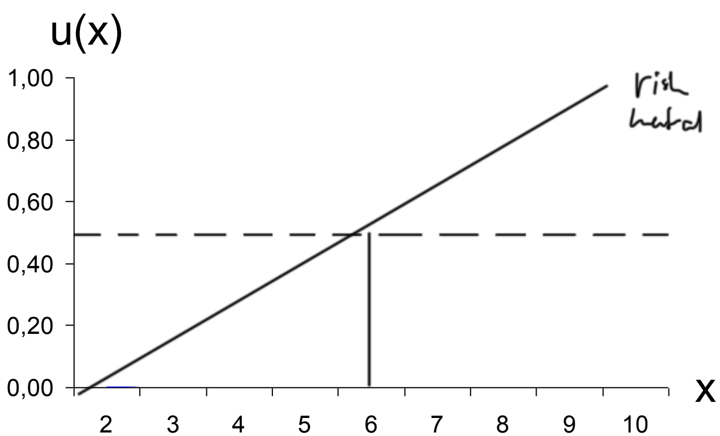
1. Building the utility function.
2. Mapping the $e_{i,j}$ of the decision matrix into utilities $u(e_{i,j})$.
3. Calculating for each action a_i the expected utility $EU(a_i)$.
4. Choosing the action a_i with maximum $EU(a_i)$.

Example:

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

Step 1: Building the Utility Function

$7 = CE? \sim$ (0.5: 10; 0.5: 2) $\rightarrow EV = 5 + 1 = 6$



Example:

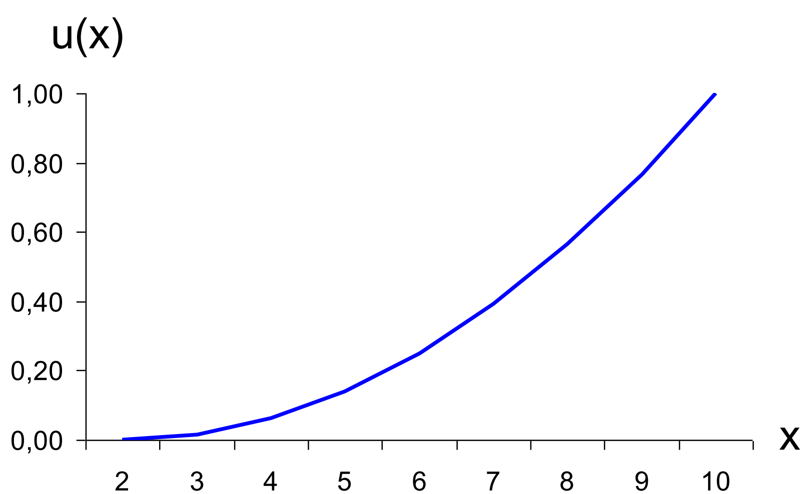
	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	2 e^-	2	6	10 e^+
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

Step 2: Mapping the Outcomes into Utilities

$e_{i,1} \Rightarrow U$ $\quad U(6) = \left(\frac{6-2}{8}\right)^2 = 0{,}25$

$e^+ = 10$
$e^- = 2$

$u(x) = \left(\frac{x-2}{8}\right)^2$ ► Risk seeking DM



	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

	p_j	0.1	0.2	0.5	0.2	$EU(a_i)$
	s_j	1	2	3	4	
a_i	1					
	2					
	3					
	4					

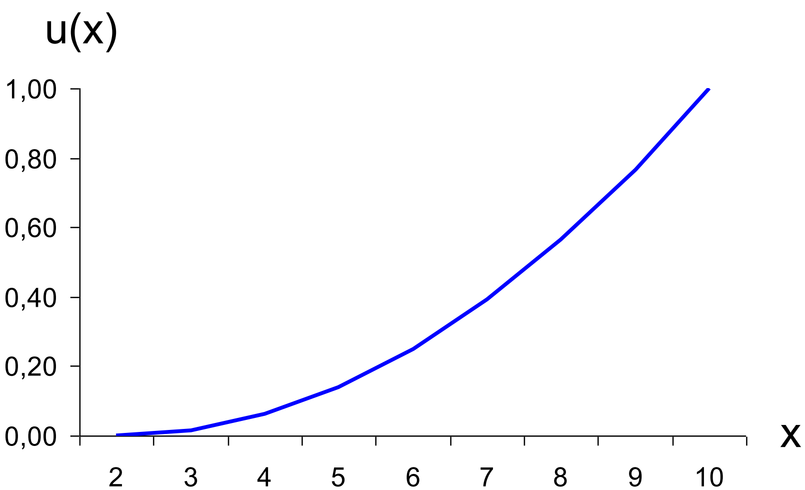
Step 3: Calculating Expected Utilities

$EU(a_i) = \sum_{j=1}^{n} \left(p_j \cdot u(e_{i,j}) \right)$

$e^+ = 10$

$e^- = 2$

$u(x) = \left(\frac{x-2}{8} \right)^2$



$EU(a_1) = 0.1 \cdot 0 + 0.2 \cdot 0 + 0.3 \cdot 0.25 + 0.4 \cdot 1 = 0.33$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

	p_j	0.1	0.2	0.5	0.2	$EU(a_i)$
	s_j	1	2	3	4	
a_i	1	0	0.00	0.25	1	0.33 highest expected utility
	2	0.25	0.02	0.14	0.06	0.11
	3	0.06	0.56	0.06	0.14	0.18
	4	0.02	0.77	0.14	0	0.23

1.4 Decision Making under Risk: The μ-σ Rule

μ-σ Rule: Building Blocks

$\mu(a_i)$: Expected outcome of action a_i

$\mu(a_i) = EV(a_i) =$

$\sigma^2(a_i)$: Variance of the outcome of action a_i

$\sigma^2 (a_i) =$

$\sigma(a_i)$: Standard deviation of the outcome of action a_i

$\sigma(a_i) =$

μ-σ Building Blocks: Example

	p_j	0.1	0.2	0.5	0.2	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
	s_j	1	2	3	4			
a_i	1	2	2	6	10			
	2	6	3	5	4			
	3	4	8	4	5			
	4	3	9	5	2			

μ-σ Rule: Preference Function

$\Phi(a_i)$: Preference function

$\Phi(a_i) = f(\mu(a_i),\sigma(a_i))$

	Risk neutral DM	Risk seeking DM	Risk averse DM
Generic:	$\Phi(a_i) = \mu_i(a_i)$	$\Phi(a_i) = \mu(a_i) +$	
Example:			

μ-σ Rule for a Risk Neutral Decision Maker

Preference function: $\Phi(a_i) = \mu(a_i)$

	p_j	0.1	0.2	0.5	0.2	$\mu(a_i)$	$\sigma^2(a_i)$	$\Phi(a_i)$
	s_j	1	2	3	4			
a_i	1	2	2	6	10	5.6	7.8	
	2	6	3	5	4	4.5	0.9	
	3	4	8	4	5	5	2.4	
	4	3	9	5	2	5	5.4	

μ-σ Rule for a Risk Seeking Decision Maker

Preference function: $\Phi(a_i) = \mu(a_i)+0.5\cdot\sigma^2(a_i)$

							$\mu(a_i)$	$\sigma^2(a_i)$	$\Phi(a_i)$
	p_j	0.1	0.2	0.5	0.2				
	s_j	1	2	3	4				
a_i	1	2	2	6	10		5.6	7.8	
	2	6	3	5	4		4.5	0.9	
	3	4	8	4	5		5	2.4	
	4	3	9	5	2		5	5.4	

μ-σ Rule for a Risk Averse Decision Maker

Preference function: $\Phi(a_i) = 1.5 \cdot \mu(a_i) - 2 \cdot \sigma(a_i)$

	p_j	0.1	0.2	0.5	0.2	$\mu(a_i)$	$\sigma^2(a_i)$	$\Phi(a_i)$
	s_j	1	2	3	4			
a_i	1	2	2	6	10	5.6	7.8	
	2	6	3	5	4	4.5	0.9	
	3	4	8	4	5	5	2.4	
	4	3	9	5	2	5	5.4	

Expected Utility vs. μ-σ Rule

μ-σ rule and expected utility will lead to the same decision in case of

- Risk neutral decision maker
- Quadratic utility function and appropriate calibration of preference function $\Phi(x)$
- Normally distributed outcomes

Drawbacks of the μ-σ rule:

► Does not guide the decision maker in calibrating the preference function $\Phi(x)$.

► Cannot determine the risk attitude of the decision maker.

1.5 Decision Trees

Characterization of the Decision Context

- Risk
- Single goal
- Single decision maker
- Dynamic scenarios

Decision matrix

Decision tree

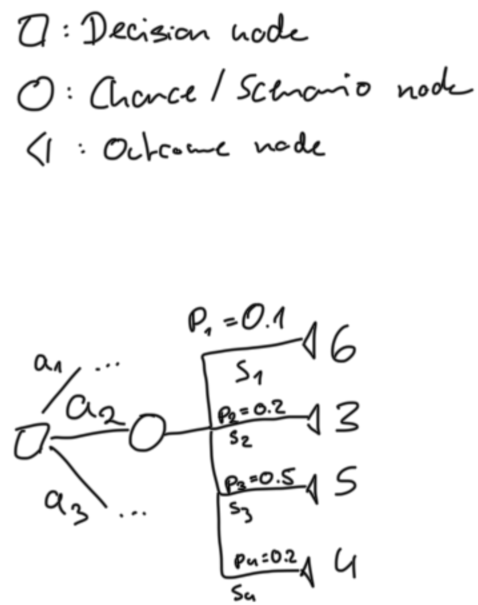
→ Repeated sequence of action and occurrence of a scenario

From Decision Matrix to Decision Tree: Single Action

Decision matrix

	p_j	0,1	0,2	0,5	0,2
	s_j	1	2	3	4
	1	2	2	6	10
	2	6	3	5	4
a_i	3	4	8	4	5
	4	3	9	5	2
	5	4	4	4	4

Decision tree

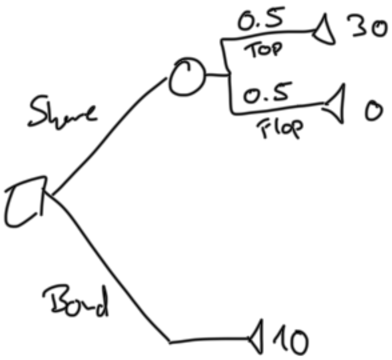


From Decision Matrix to Decision Tree: All Actions

Decision matrix

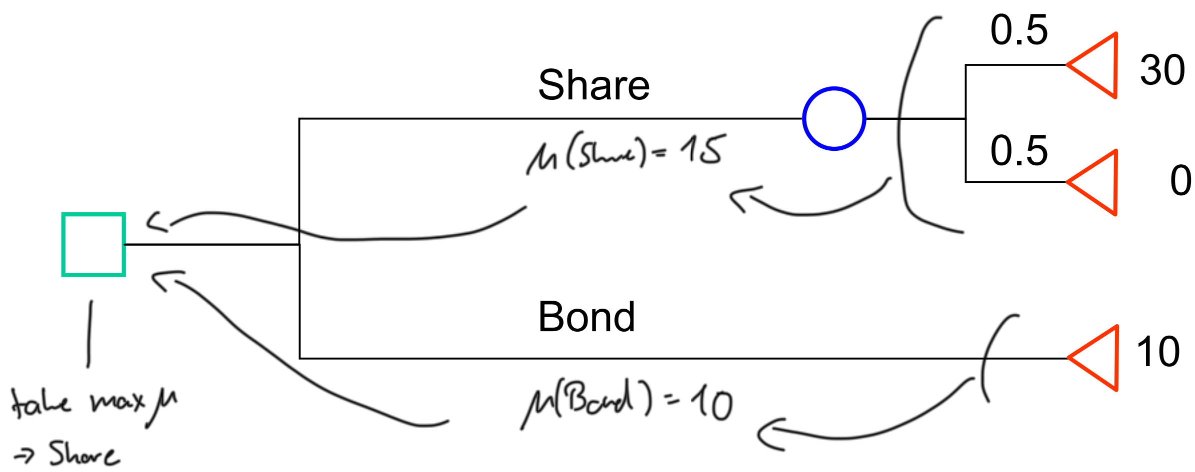
	s_1: Top p_1= 0.5	s_2: Flop p_2= 0.5
a_1: Share	30K	0
a_2: Bond	10K	10K

Decision tree



Solving the Decision Tree: Roll Back Procedure

Assumption: Risk neutral decision maker → $\Phi(a_i)=\mu(a_i)$ (μ-principle)



Decision Tree: Fully Fledged Example

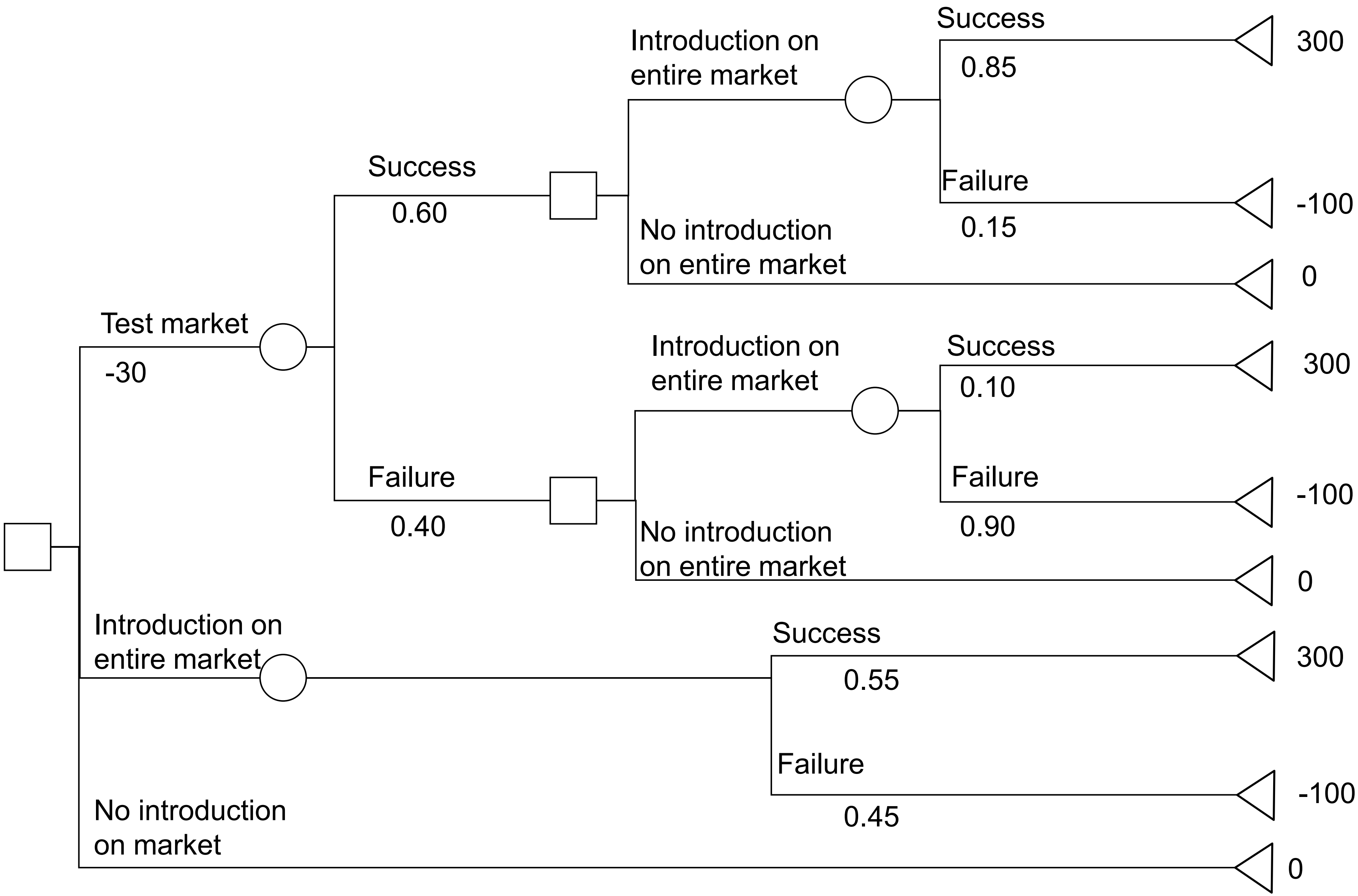
- Introduction of a new product
  - Market success: 300 profit
  - Market failure: 100 loss

- Action 1: First test market, then introduction on the entire market
  - Costs test market 30
  - A priori success probability for test market 60%
  - In case test market is a success: 85% success probability for entire market
  - In case test market is a failure: 10% success probability for entire market

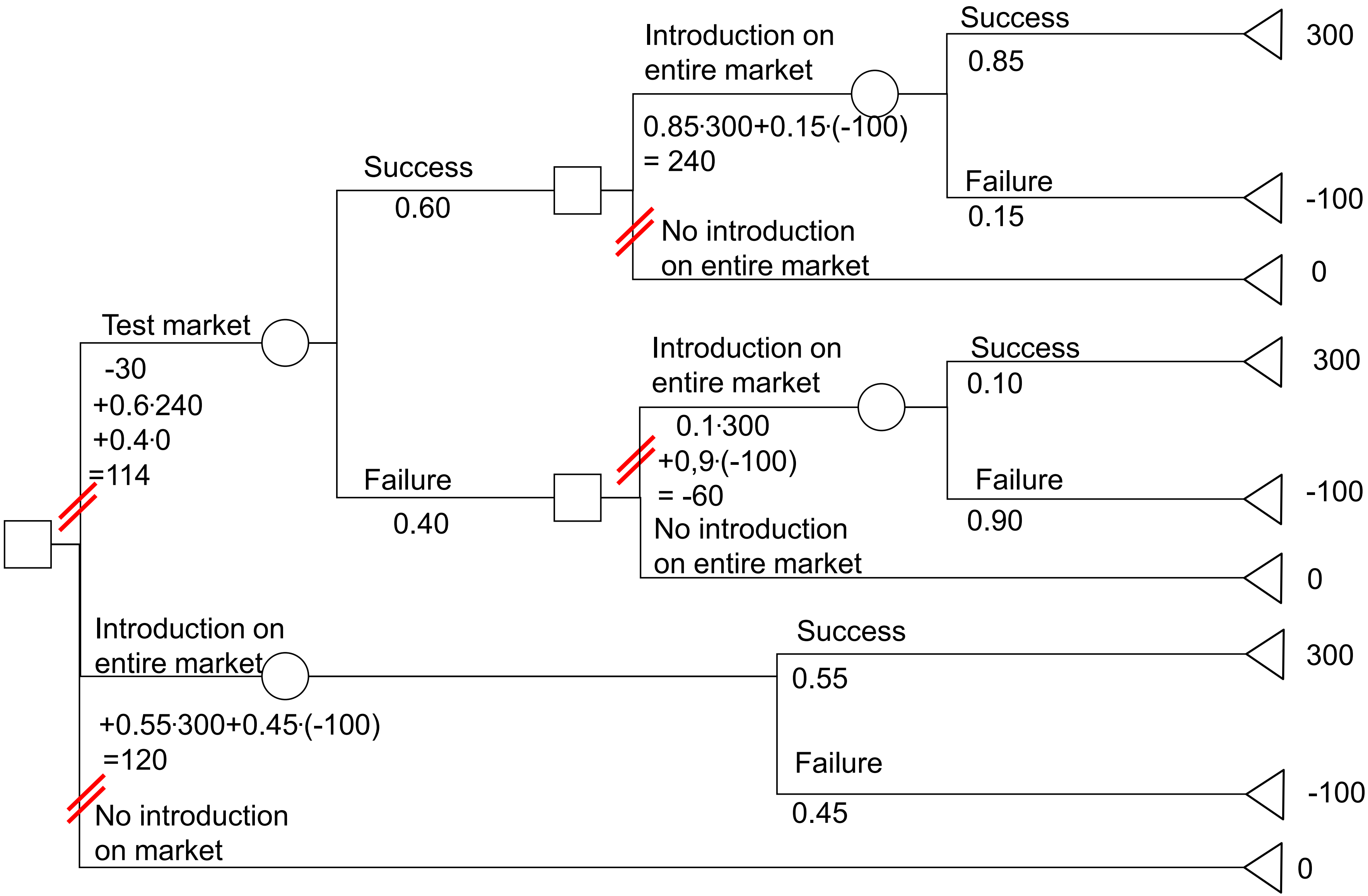
- Action 2: Market introduction without test market
  - 55% success probability

- Action 3: No introduction on the market

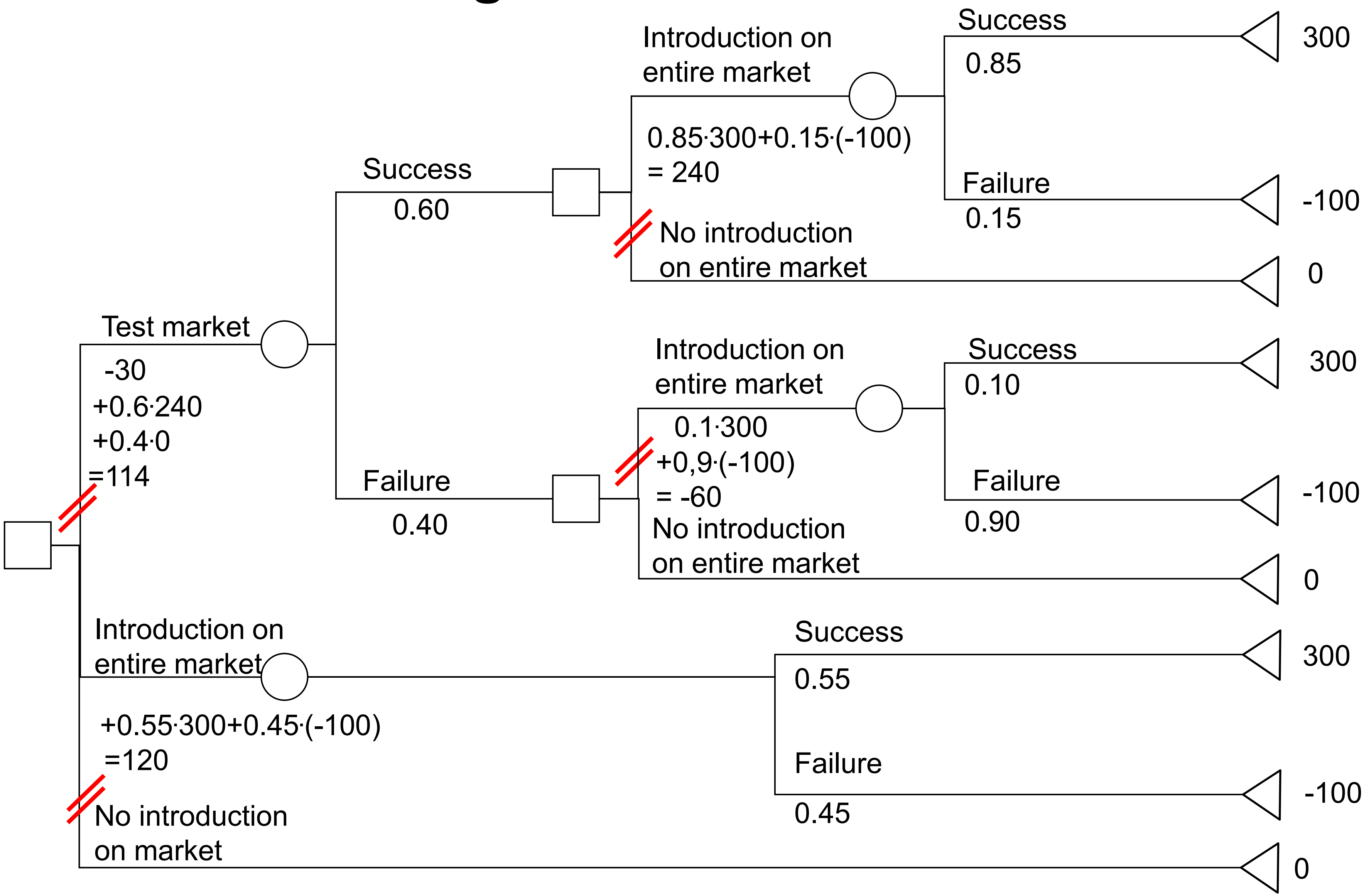
Solving the Decision Tree



Optimal Solution of the Decision Tree



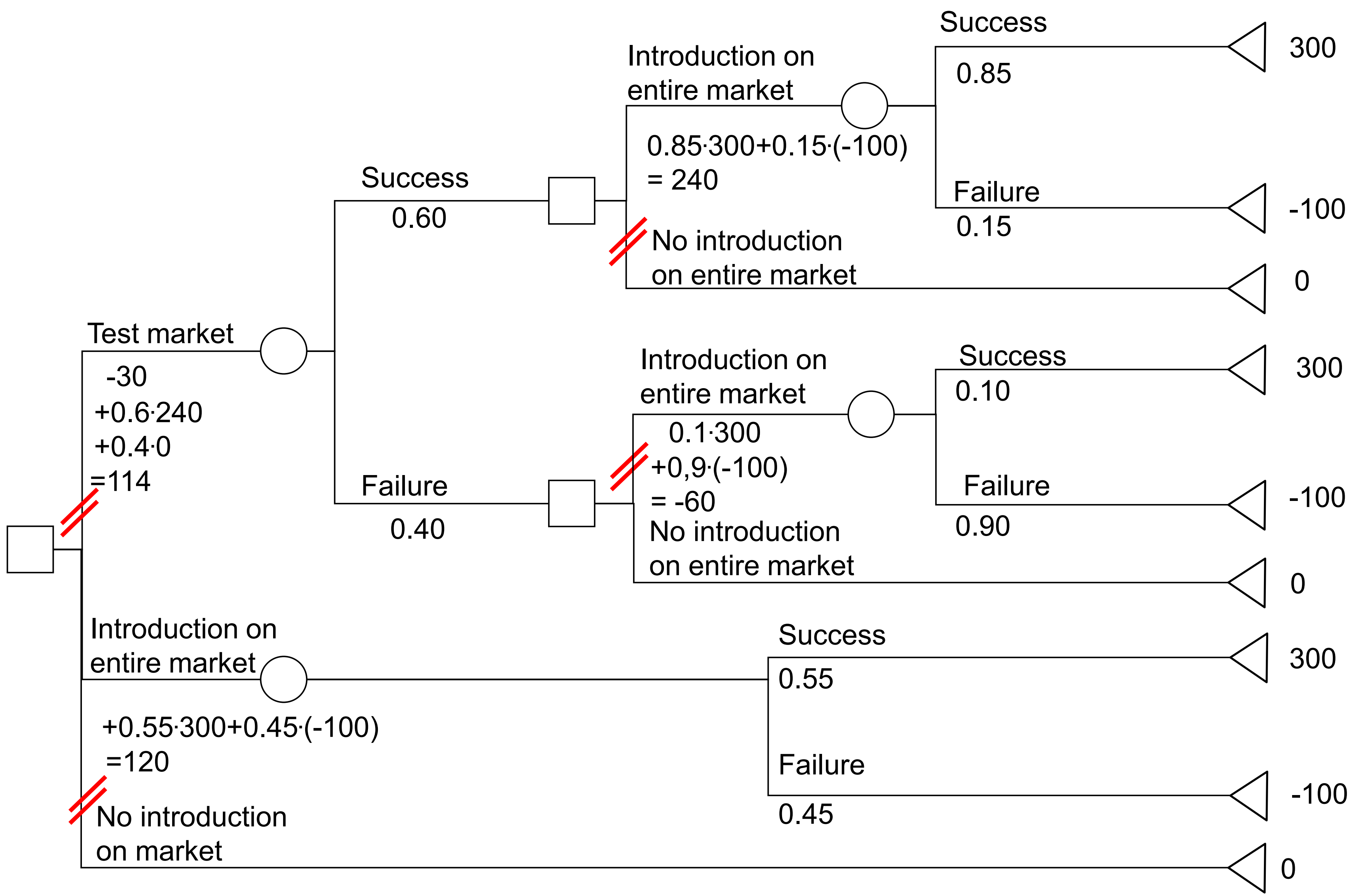
Flexible Planning



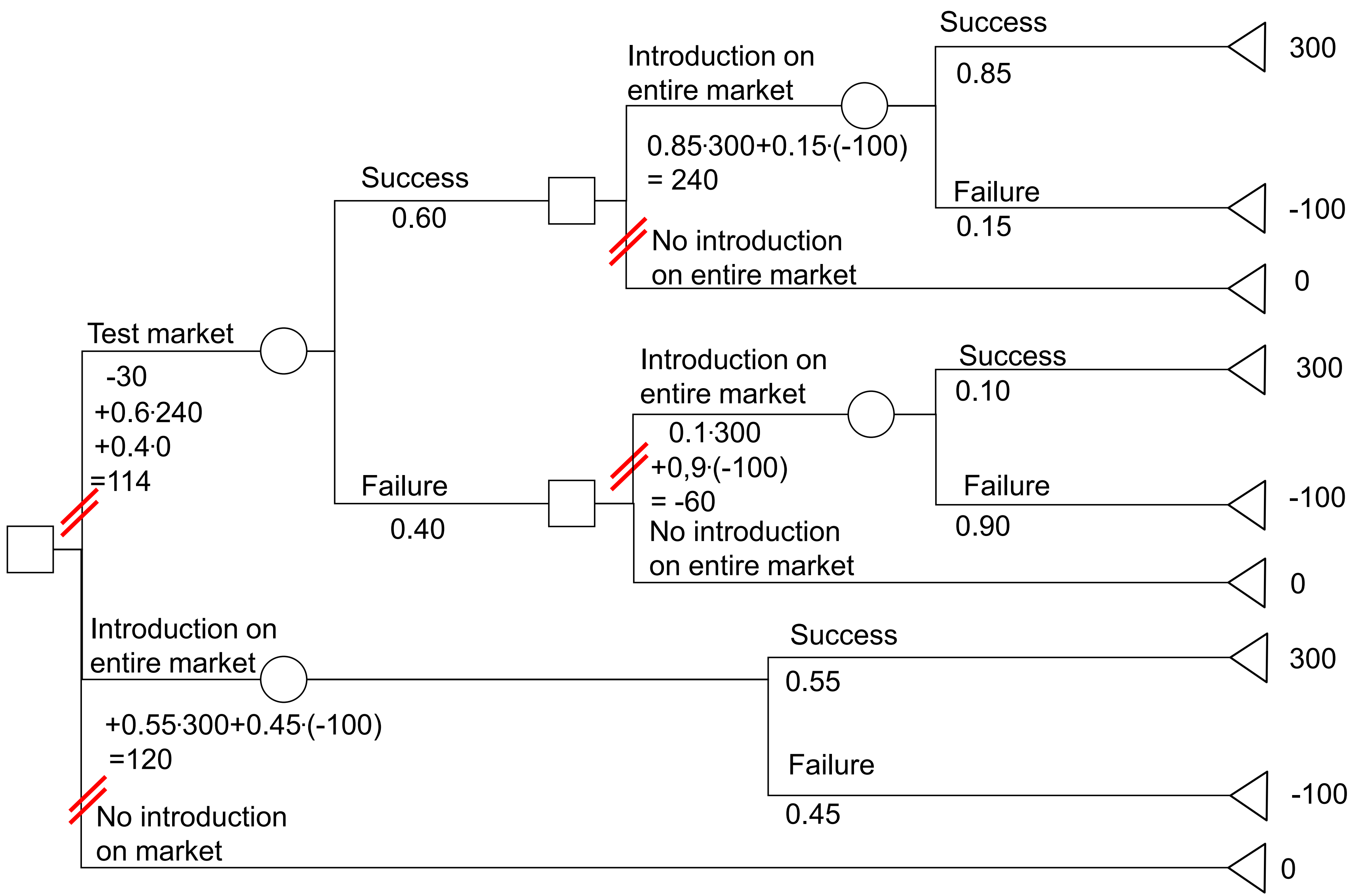
Valuation of the Test Market

- The result of the test market provides better information about the probability of success on the overall market.
- What is the value of this information?

Decision Tree with Free Test Market



Decision Tree without Test Market



Value of the Test Market

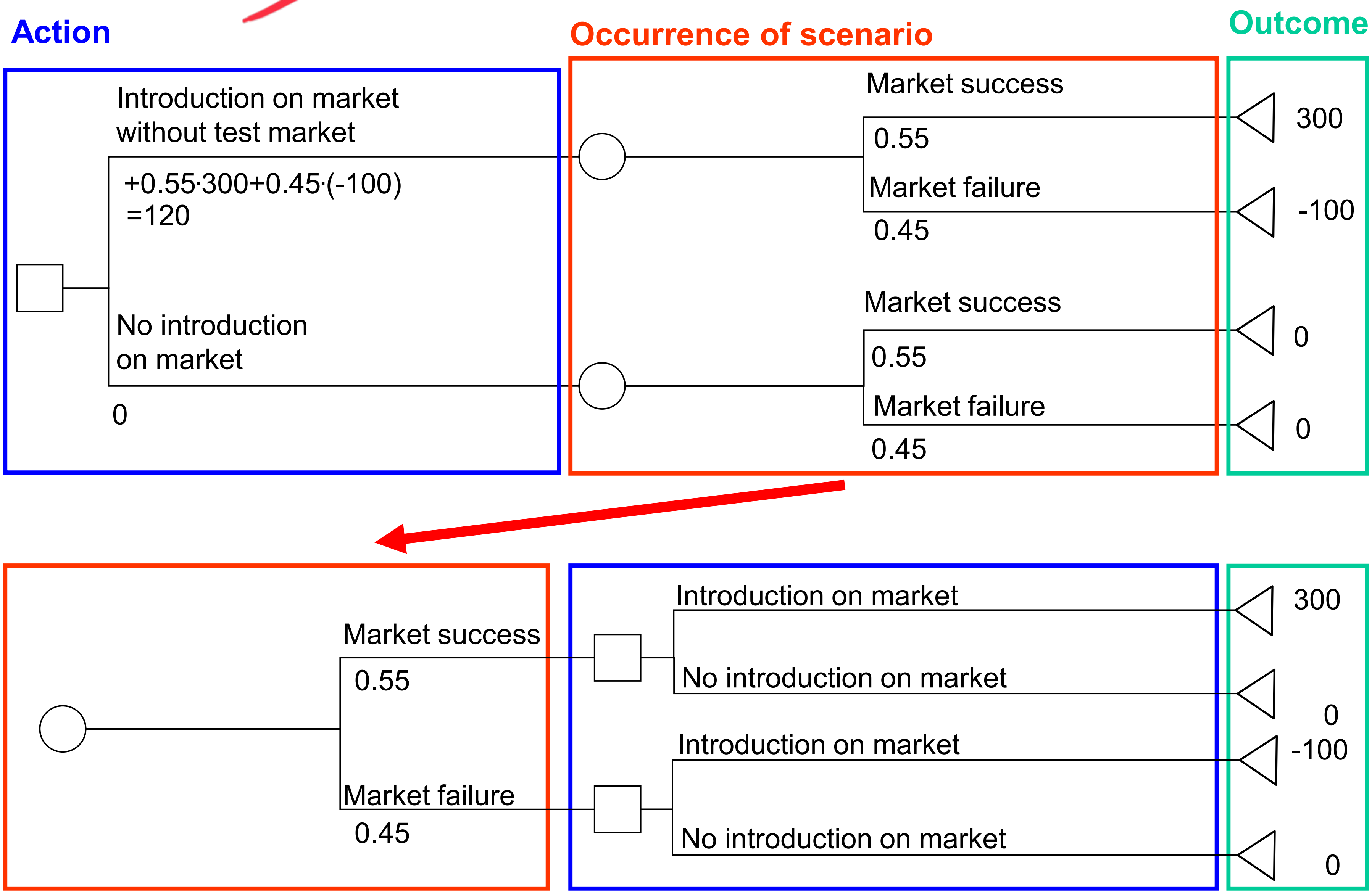
	Value with test market free of cost	=
-	Value without test market	=
	Value of test market	=

Value of Perfect Information

Assumptions:

- The sequence of action and occurrence of a scenario is reversed in the decision tree.
- The DM can observe first, which scenario takes place and then choose an action.
- Perfect information does not mean that there will be market success in any case.
- The probability of 55% for the market success in case of no test market does not change.

Decision Tree with Perfect Information



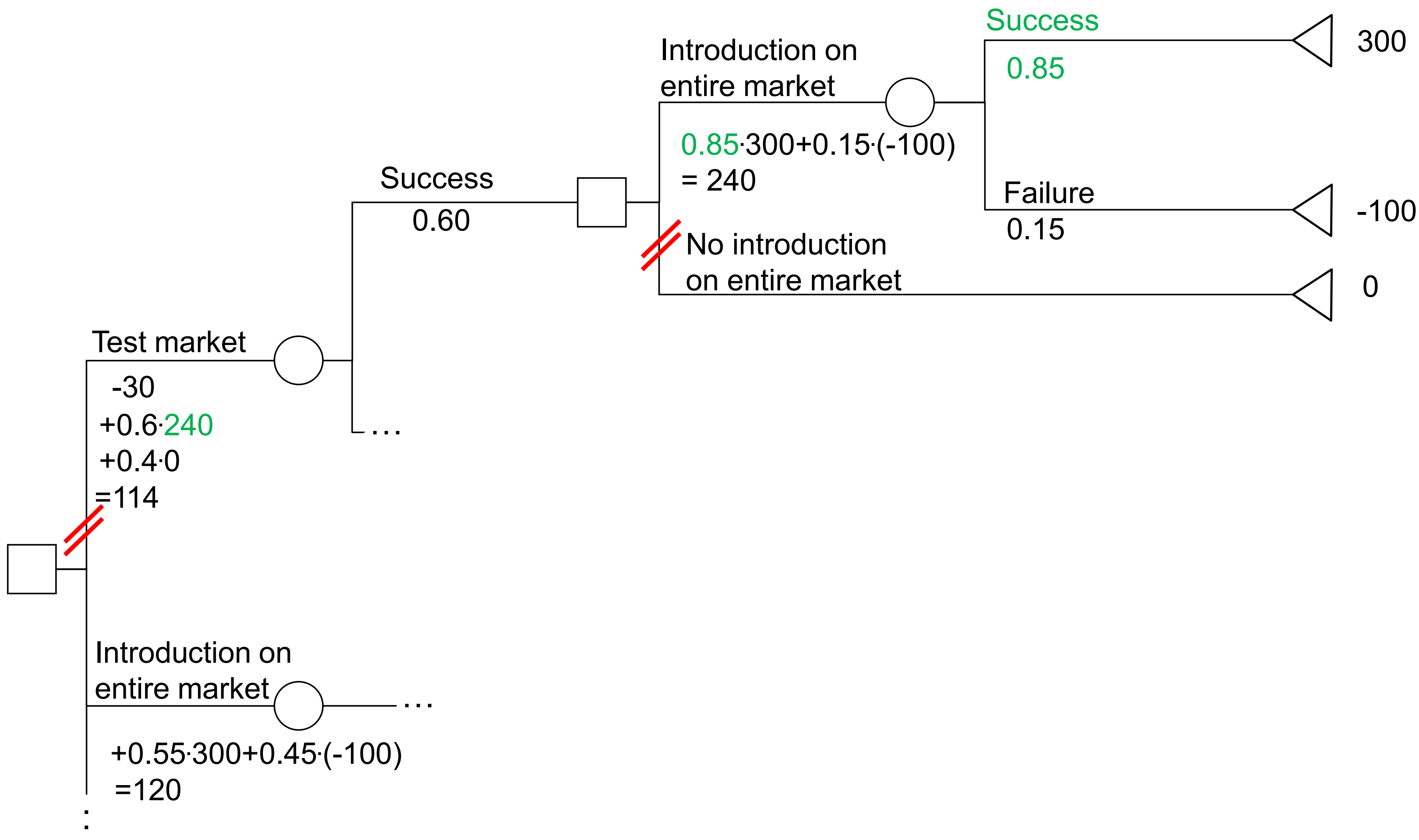
Value of Perfect Information

Value with perfect information =
- Value without perfect information =

Value of perfect information =

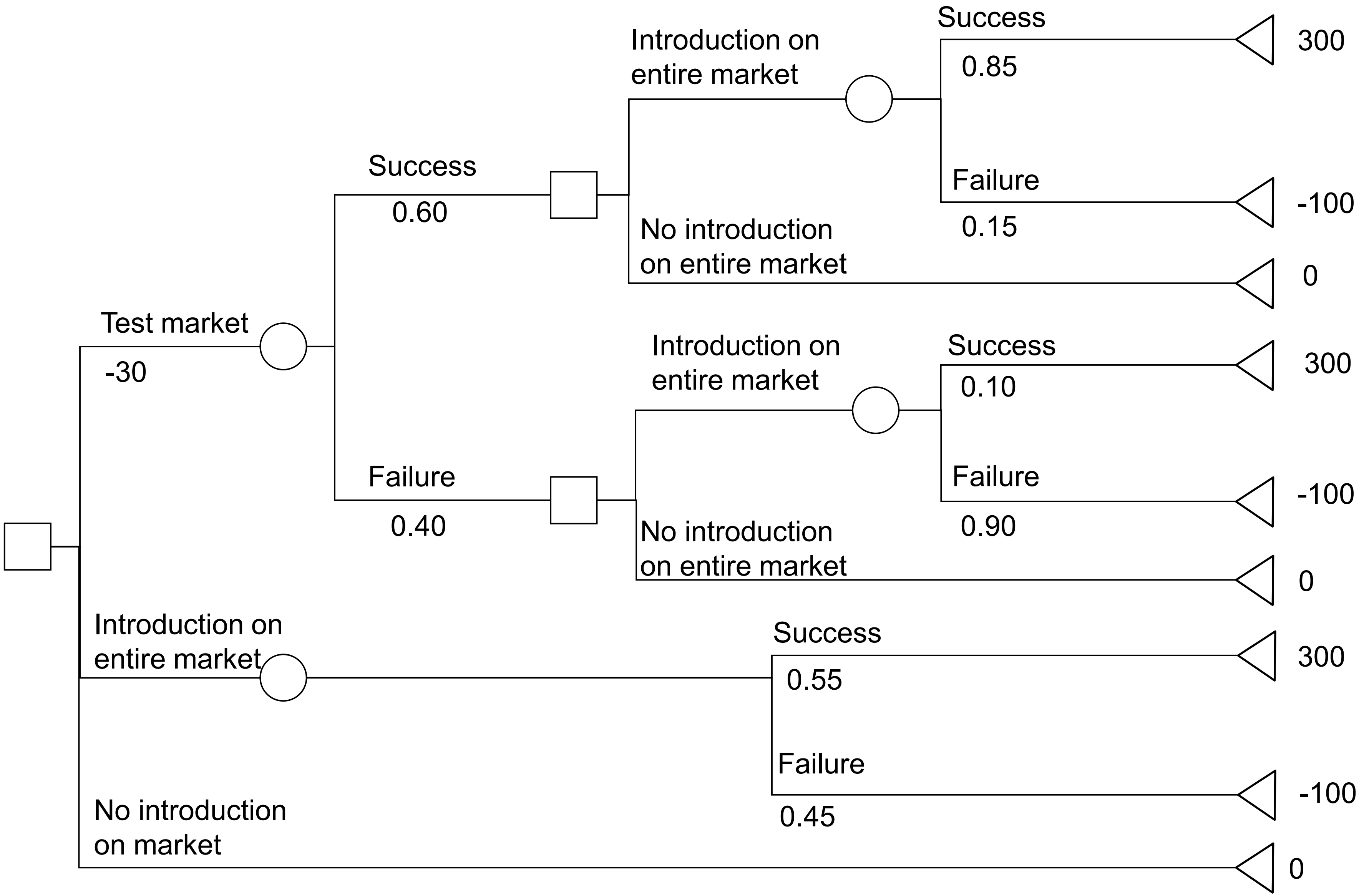
Sensitivity Analysis

Question: How does a change of the forecast accuracy of the TM impacts the decision ?



Solving the Decision Tree with Expected Utility

Step 1: Shifting profits and costs to the outcome nodes

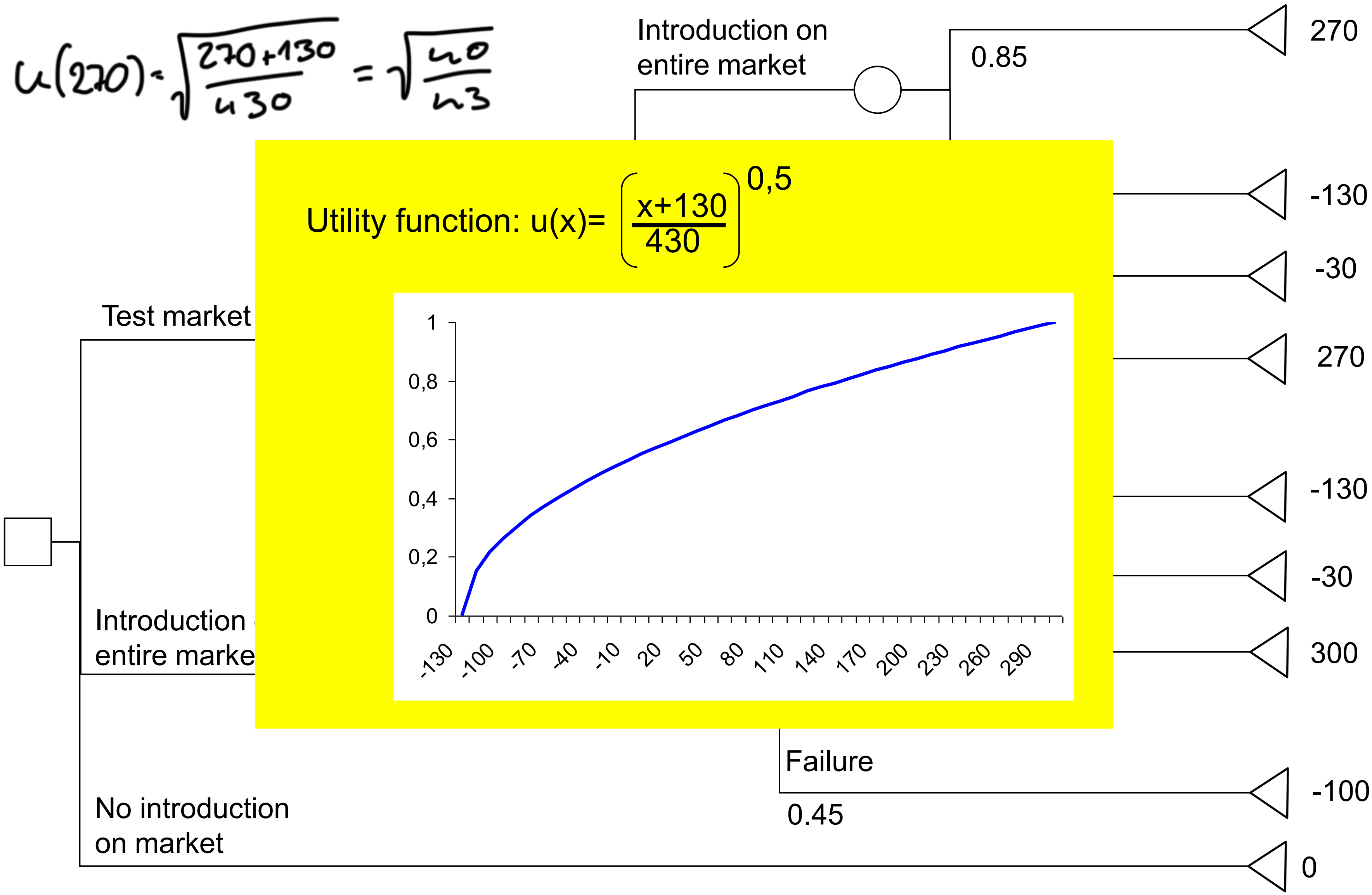


Solving the Decision Tree with Expected Utility

Step 2: Deriving the Utility Function

$$u(270) = \sqrt{\frac{270+130}{430}} = \sqrt{\frac{40}{43}}$$

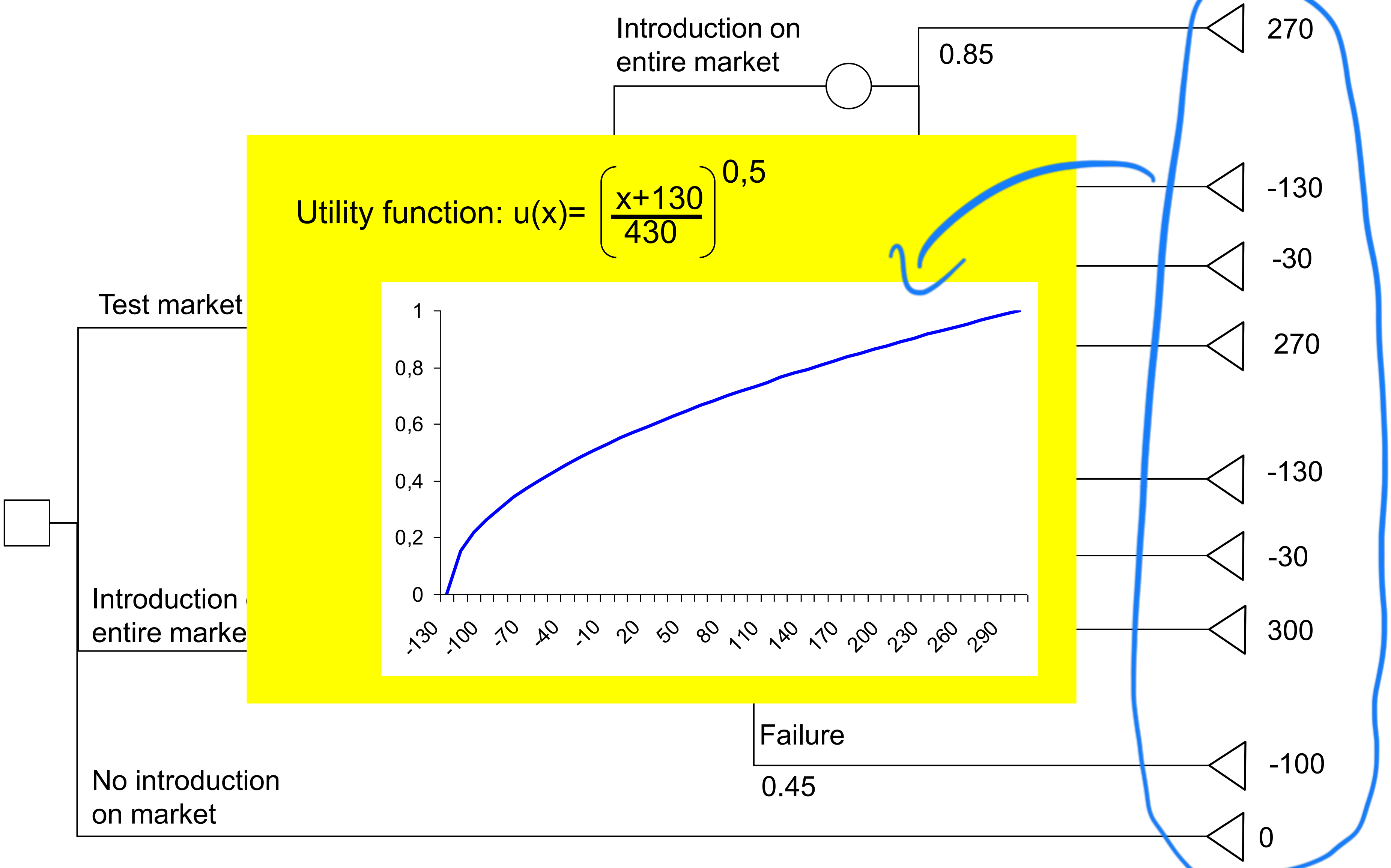
Utility function: $u(x) = \left(\frac{x+130}{430}\right)^{0,5}$



Solving the Decision Tree with Expected Utility

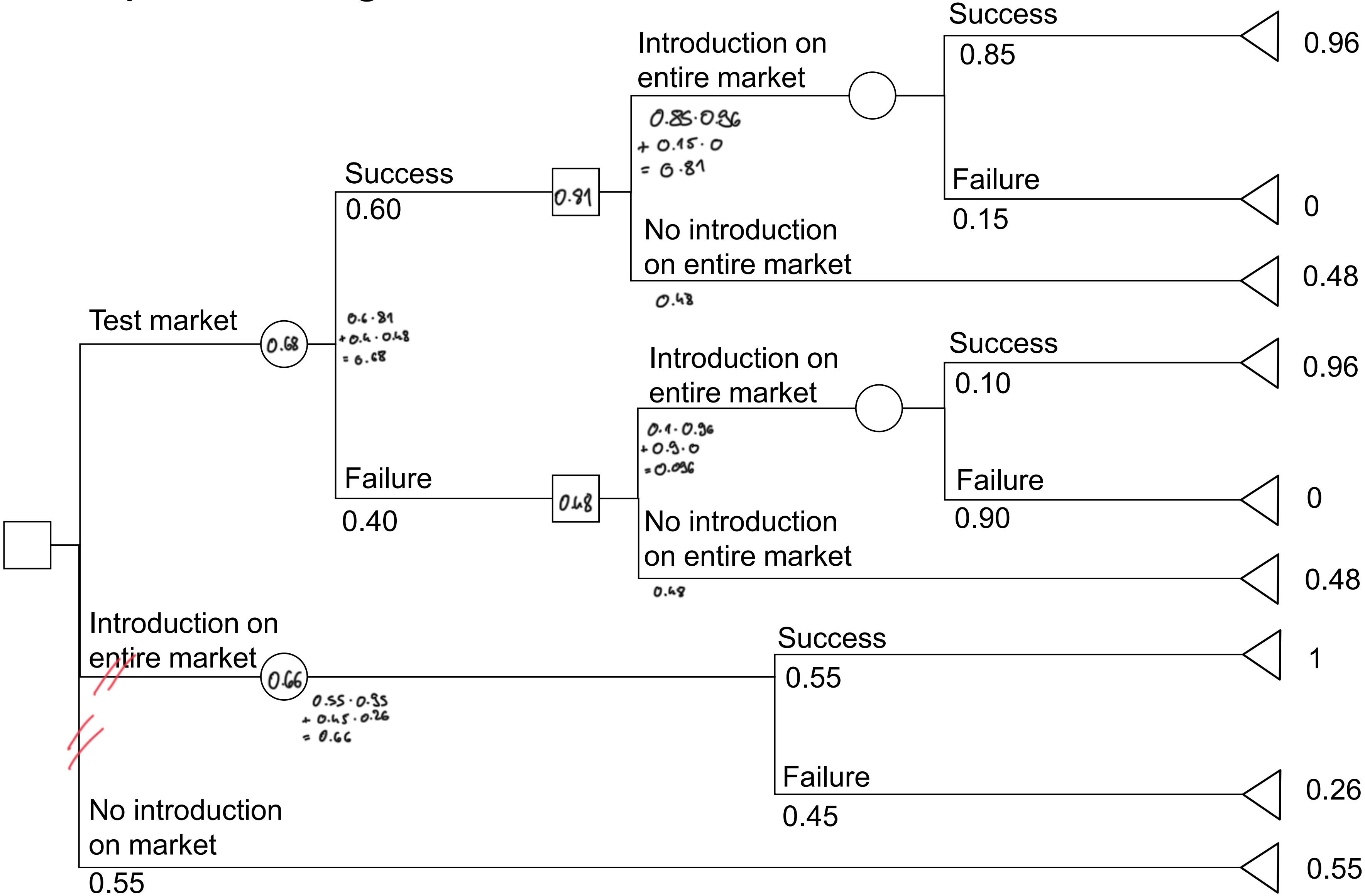
Step 3: Transforming outcomes x in utilities $u(x)$

Utility function: $u(x) = \left(\frac{x+130}{430}\right)^{0,5}$



Solving the Decision Tree with Expected Utility

Step 4: Solving the Decision Tree with Roll Back



1.6 Multi-Criteria Decision Making: The Scoring Model

Multi-Criteria Decision Making: Characterization of the Decision Context

Characterization:

- Deterministic
- Multiple goals
- Single decision maker
- Static

Methods:

- Scoring Modell (“Nutzwertanalyse”)
- Analytical Hierarchy Process (AHP)
- Multi-Attribute Utility Theory
- …

Modification of the Decision Matrix

	z_1	…	z_j	…	z_n
	w_1	…	w_j	…	w_n
a_1	$e_{1,1}$	…	$e_{1,j}$	…	$e_{1,n}$
…					
a_i	$e_{i,1}$	…	$e_{i,j}$	…	$e_{i,n}$
…					
a_m	$e_{m,1}$	…	$e_{m,j}$	…	$e_{m,n}$

Scoring Model: Example

	Cost w_1	Success on export markets w_2
Action 1	20	low
Action 2	30	good
Action 3	50	very good

-> Determine weights

-> Transform into scores [0,1]

Scoring Model:
Step 1: Criteria Weights

<u>Method</u>:

1. The value g_j of each goal z_j is determined according to the following scale:

Very important:	5
Important:	4
Average important:	3
Less important:	2
Marginally important:	1

2. The relative importance w_j of each goal is determined according to:

$$w_j = \frac{g_j}{\Sigma g_i}$$

normalizes to [0,1]

<u>Example</u>:

Cost w_1	Success on export markets w_2
Costs are important	Success on export markets is less important.
$g_1 = 4$	$g_2 = 2$

$w_1 = \frac{4}{4+2} = \frac{2}{3}$

Scoring Model:

Step 2: Transforming Qualitative into Quantitative Values

	Success on export markets (qualitative)	Success on export markets (quantitative)
Action 1	low	
Action 2	good	
Action 3	very good	

very good	5
good	4
average	3
modest	2
low	1

Scoring Model:

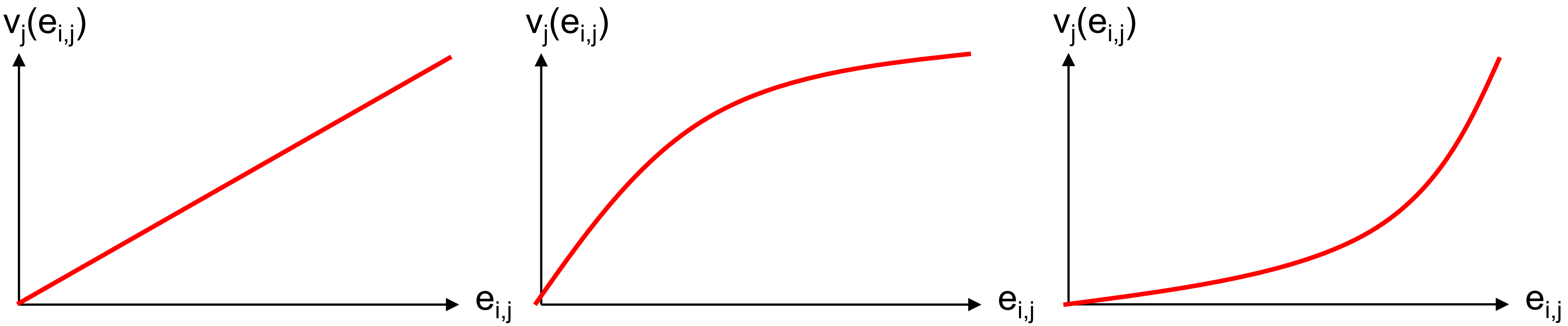
Step 3: Normalizing Scales

	Cost w_1=0.67	Success on export markets w_2=0.33	
Action 1	20	1	$Score_1 = w_1 \cdot e_{11} + w_2 \cdot e_{12} = 0.67 \cdot 20 + 0.33 \cdot 1$
Action 2	30	4	...
Action 3	50	5	...

Value Function

- Outcomes $e_{i,j}$ of actions $a_1,\ldots, a_m$ with respect to goal z_j are normalized to range [0,1] by value function $v_j(e_{i,j})$

- Characteristics of the value function:
  - monotonically increasing (decreasing)
  - worst outcome has value 0
  - best outcome has value 1

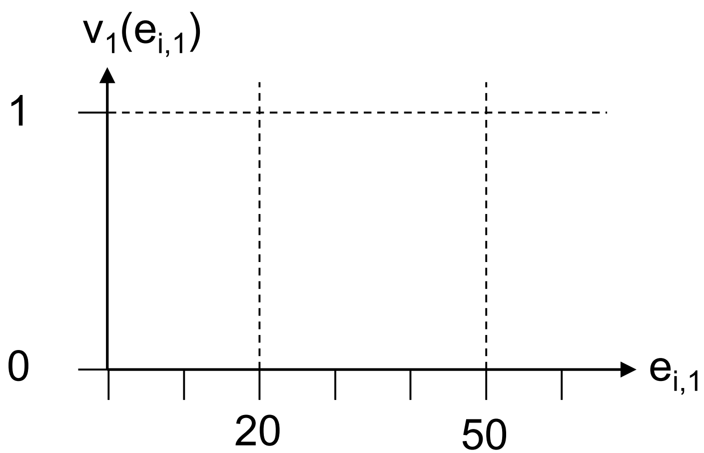


→ For the remainder we are using the linear value function

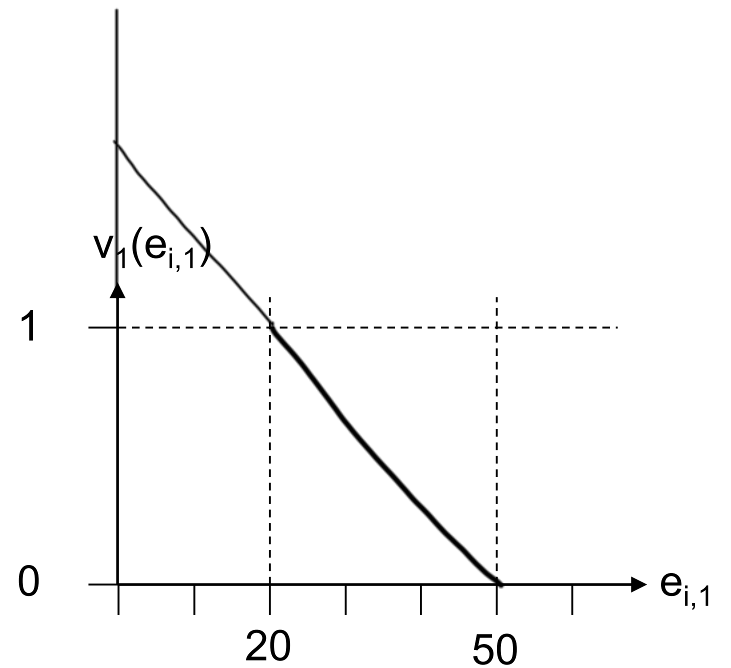
Value Function of the Cost Goal

	Cost w_1=0.67
Action 1	20
Action 2	30
Action 3	50

<u>Value function</u>:



<u>Linear value function</u>:



Value Function of the Goal Export Markets

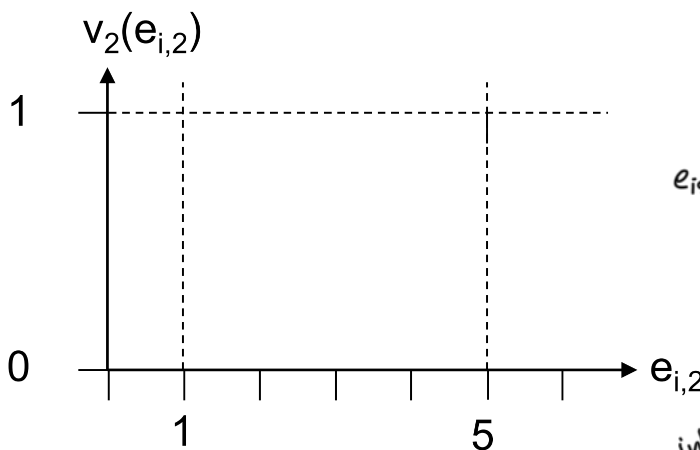
	Success on export markets	
Alternative 1	1	→ 0
Alternative 2	4	→ 0.75
Alternative 3	5	→ 1

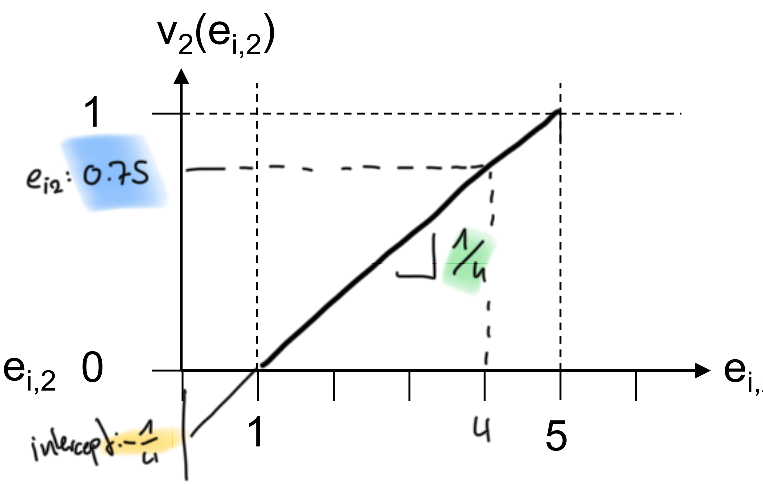
Value function:

Linear value function:

$V_2(e_{i2}) = -\frac{1}{4} + \frac{1}{4} e_{i1}$

$20 \leq e_{i1} \leq 50$





Scoring Model: Calculating Overall Scores

$$s(a_i) = \sum_{j=i}^{n} w_j \cdot v_j(e_{ij})$$

	Cost w_1=0.67	Success on export markets w_2=0.33 $\sum w_n = 1$	Score
Action 1	1 e_{11}	0 e_{12}	
Action 2	0.67 e_{21}	0.75 e_{22}	
Action 3	0 e_{31}	1 e_{32}	

$s(a_1) = 0.67 \cdot 1 + 0.33 \cdot 0 = 0.67$

$s(a_2) = 0.69$

$s(a_3) = 0.33$

Scoring Model: Sensitivity Analysis

	Cost $w_1=0.67$	Success on export markets $w_2=0.33$	Score
Action 1	1	0	0.67
Action 2	0.67	0.75	0.69
Action 3	0	1	0.33

Question:
How robust is the result with respect to the criteria weights?

$$S(a_i) = w_1 \cdot e_{i1} + (1-w_1) \cdot e_{i2}$$

Scoring Model: Sensitivity Analysis