Operations Research and Decision Analysis
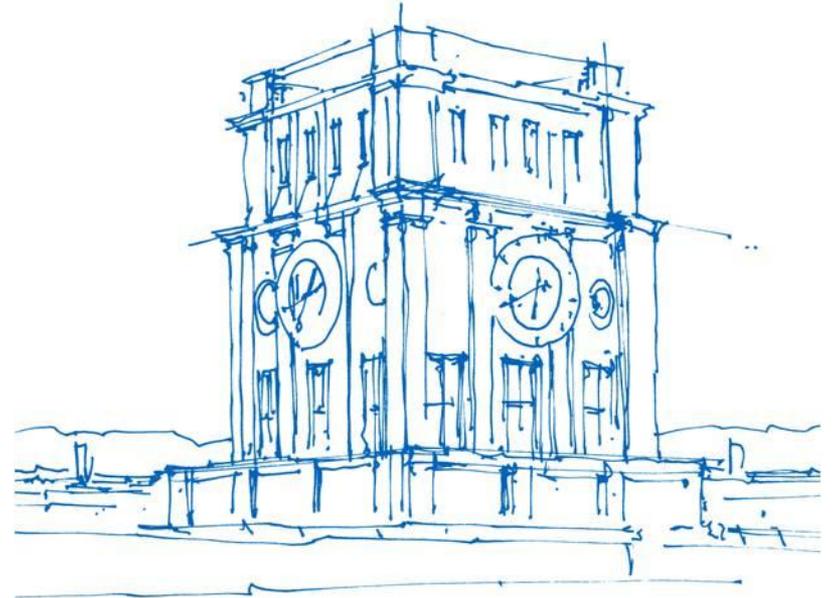# Exercise Session 1

Baturhan Bayraktar

Technical University of Munich

Chair of Operations Management

24.10.2025

Organization

21 entries available													
Semester													
Course no. ▲▼	Sem. ▲▼	Term.	Title ▲▼	Duration ▼	Type ▲▼	Part/Info	SPO C/E/O	Lecturer (Assistant) ▼	resp. org. ▲▼	Languages of Instruction ▼	Place (1st session)		Time (1st session)
Winter semester													
0000000340	W	🕐	Operations Research and Decision Analysis (MiM) Tutorials - Tutor 1 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Neumann H	TUWIB09	English	0670ZG, Hör-/Lehrsaal eben o.Exp.Bühne (0506.Z1.670)		**23.10.25** 15:00 - 16:30
0000003168	W	🕐	Operations Research and Decision Analysis (MiM) Tutorials - Tutor 1 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Neumann H	TUWIB09	English	0670ZG, Hör-/Lehrsaal eben o.Exp.Bühne (0506.Z1.670)		**23.10.25** 11:30 - 13:00
0301241662	W	🕐	Operations Research and Decision Analysis (MiM) Tutorials - Tutor 2 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Cakar S	TUWIB09	English	0534, Kleiner Hörsaal (0505.EG.534)		**23.10.25** 16:45 - 18:15
0000003760	W	🕐	Operations Research and Decision Analysis (MiM) Tutorials - Tutor 2 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Cakar S	TUWIB09	English	0540, Kleiner Hörsaal (0505.EG.540)		**24.10.25** 13:15 - 14:45
0006789762	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 1 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Martens M	TUWIB09	German	0540, Kleiner Hörsaal (0505.EG.540)		**20.10.25** 08:00 - 09:30
0000001082	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 1 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Martens M	TUWIB09	German	0670ZG, Hör-/Lehrsaal eben o.Exp.Bühne (0506.Z1.670)		**20.10.25** 09:45 - 11:15
0000089762	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 2 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Klopotowski L	TUWIB09	English	0540, Kleiner Hörsaal (0505.EG.540)		**23.10.25** 08:00 - 09:30
0123401662	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 2 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Klopotowski L	TUWIB09	English	1503, Seminarraum (0505.01.503)		**22.10.25** 08:00 - 09:30
0000003564	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 3 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Brandt L	TUWIB09	English	0534, Kleiner Hörsaal (0505.EG.534)		**24.10.25** 16:45 - 18:15
0000003299	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 3 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Brandt L	TUWIB09	English	0534, Kleiner Hörsaal (0505.EG.534)		**24.10.25** 15:00 - 16:30
0000000929	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 4 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Mayr D	TUWIB09	English	0534, Kleiner Hörsaal (0505.EG.534)		**21.10.25** 08:00 - 09:30
0000000931	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 4 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Mayr D	TUWIB09	English	1503, Seminarraum (0505.01.503)		**20.10.25** 11:30 - 13:00
0000341093	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 5 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Oberhuber S	TUWIB09	German	1503, Seminarraum (0505.01.503)		**21.10.25** 08:00 - 09:30
0000000097	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 5 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Oberhuber S	TUWIB09	German	1503, Seminarraum (0505.01.503)		**21.10.25** 09:45 - 11:15
0000003927	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 6 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Juchmes F	TUWIB09	English	0670ZG, Hör-/Lehrsaal eben o.Exp.Bühne (0506.Z1.670)		**22.10.25** 13:15 - 14:45
0000003272	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 6 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Juchmes F	TUWIB09	English	0534, Kleiner Hörsaal (0505.EG.534)		**23.10.25** 13:15 - 14:45
0000000038	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 7 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Speigl B	TUWIB09	English	2100, Seminarraum (0501.02.100)		**21.10.25** 15:00 - 16:30
0200000909	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 7 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Speigl B	TUWIB09	English	0540, Kleiner Hörsaal (0505.EG.540)		**23.10.25** 18:30 - 20:00
0000003759	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 8 - Session 1 ☆	2	TT	⊕👤		Bayraktar B [L], Juchelka J	TUWIB09	German	0534, Kleiner Hörsaal (0505.EG.534)		**20.10.25** 16:45 - 18:15
0000441099	W	🕐	Operations Research and Decision Analysis Tutorials - Tutor 8 - Session 2 ☆	2	TT	⊕👤		Bayraktar B [L], Juchelka J	TUWIB09	German	0534, Kleiner Hörsaal (0505.EG.534)		**20.10.25** 18:30 - 20:00

Agenda

1	Lecture Recap: Decision Theory
2	Exercise: Question 3
3	Exercise: Exam Winter 14/15
4	Exercise: Exam Summer 16
5	Exercise: Question 8

What is Operations Research?

What is Operations Research?

"Operations Research is a discipline that deals with the application of advanced analytical methods to help make better decisions."

Management Science is the <u>business use</u> of Operations Research
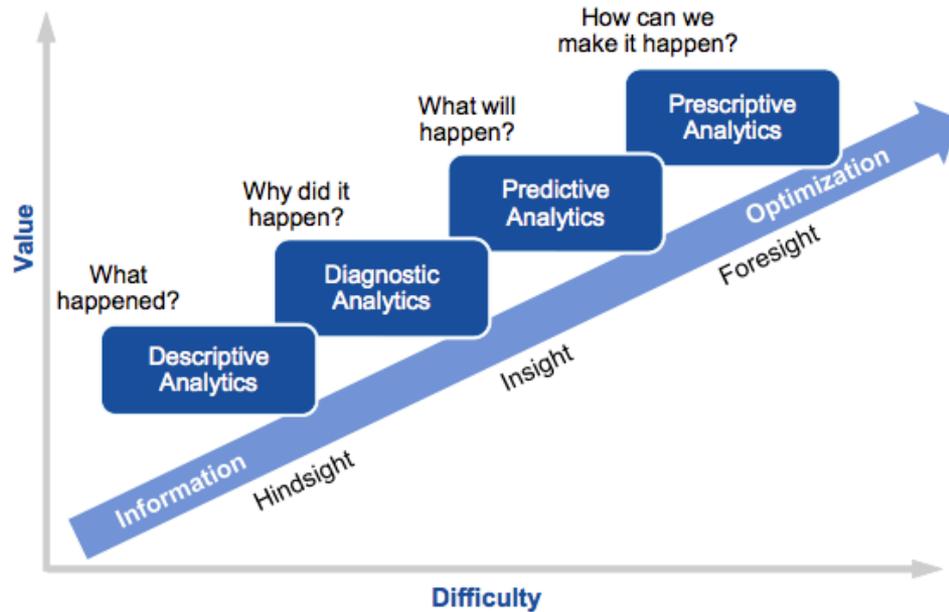
What is Operations Research?

Source: Gartner (March 2012)

What is Operations Research?

Source: Gartner (March 2012)

What is Operations Research?

Number of Registered Cars in Different Engine Types in Germany

Statistical Analysis

Source: Gartner (March 2012)

What is Operations Research?

Source: Gartner (March 2012)

What is Operations Research?

Source: Gartner (March 2012)

Optimization, „Decision Making"

Decisions!!!

- How many products should I produce?
- Should I advertise my new products?
- Should I increase my production capacity?
- Depending on the several scenarios, how should I act now and reoptimize in the future?

What is Operations Research?

Source: Gartner (March 2012)

Optimization, „Decision Making"
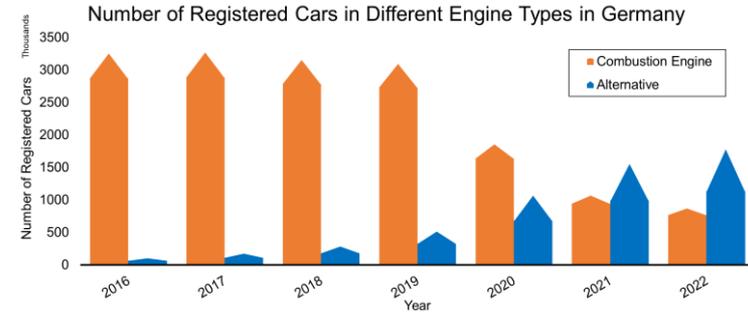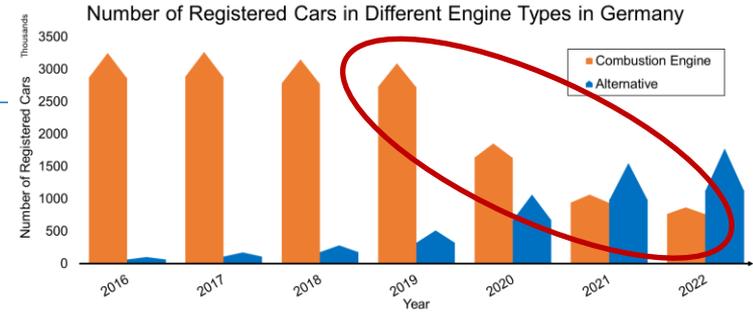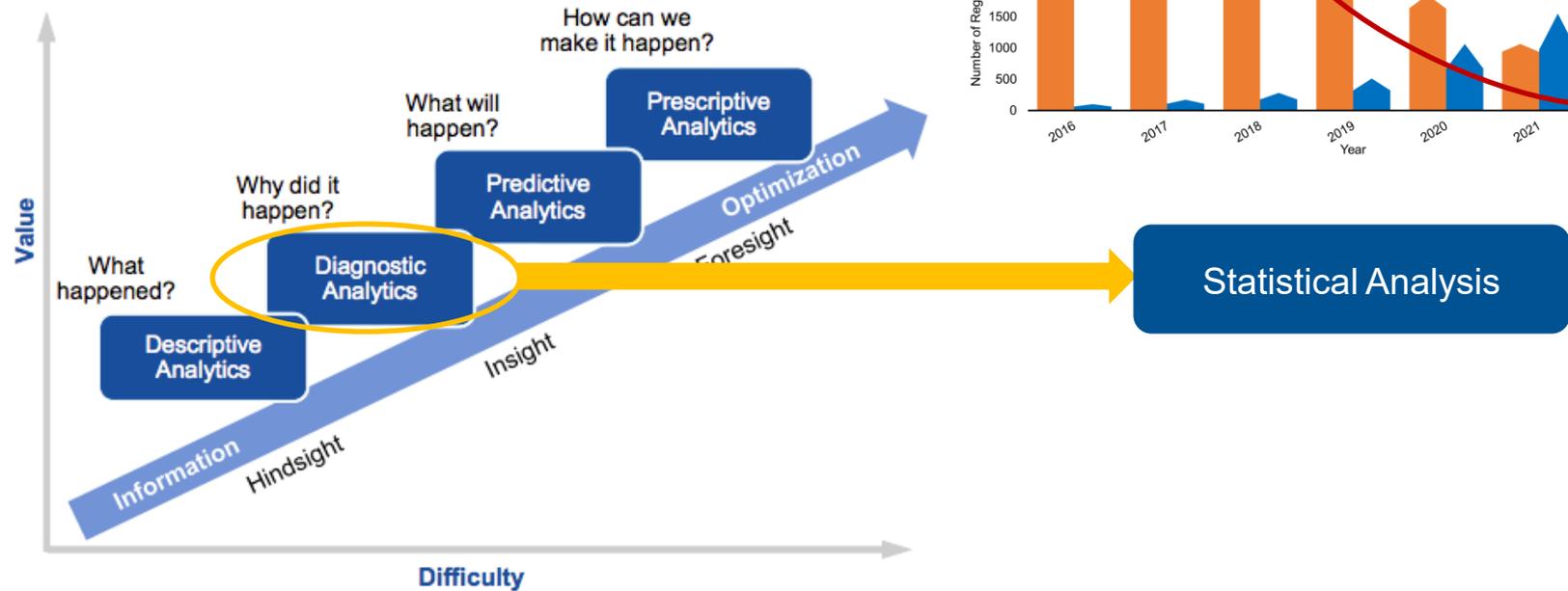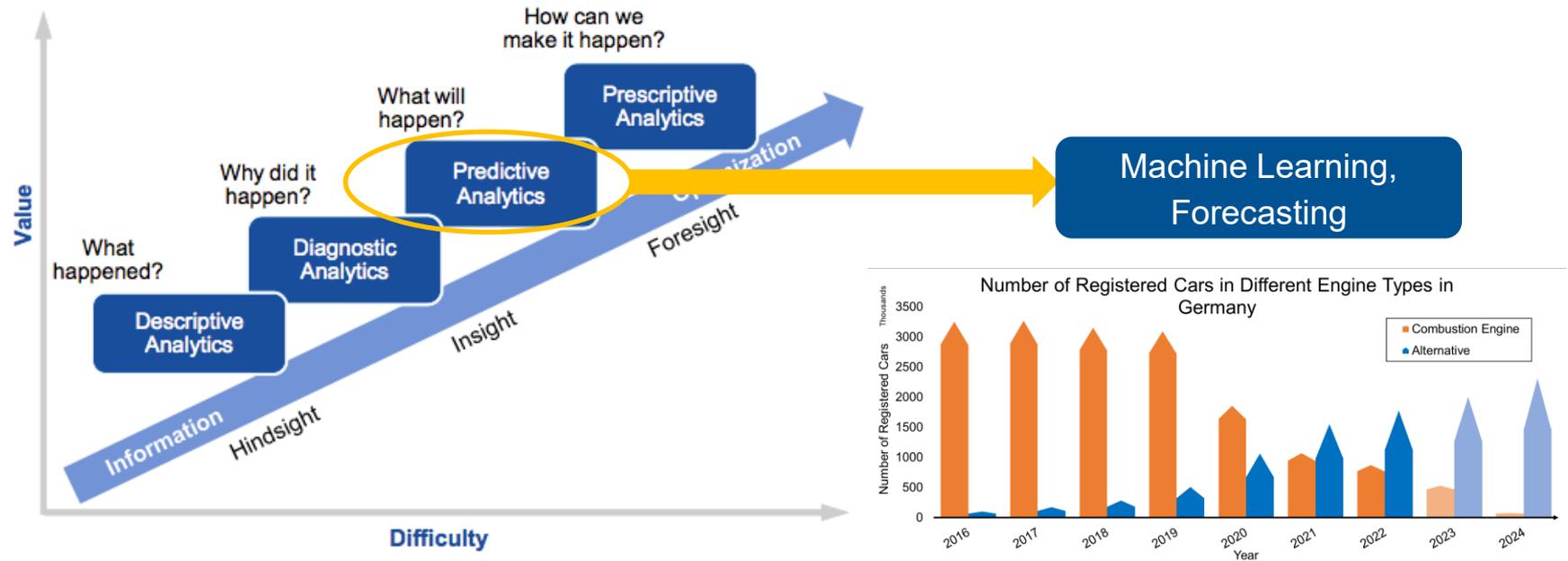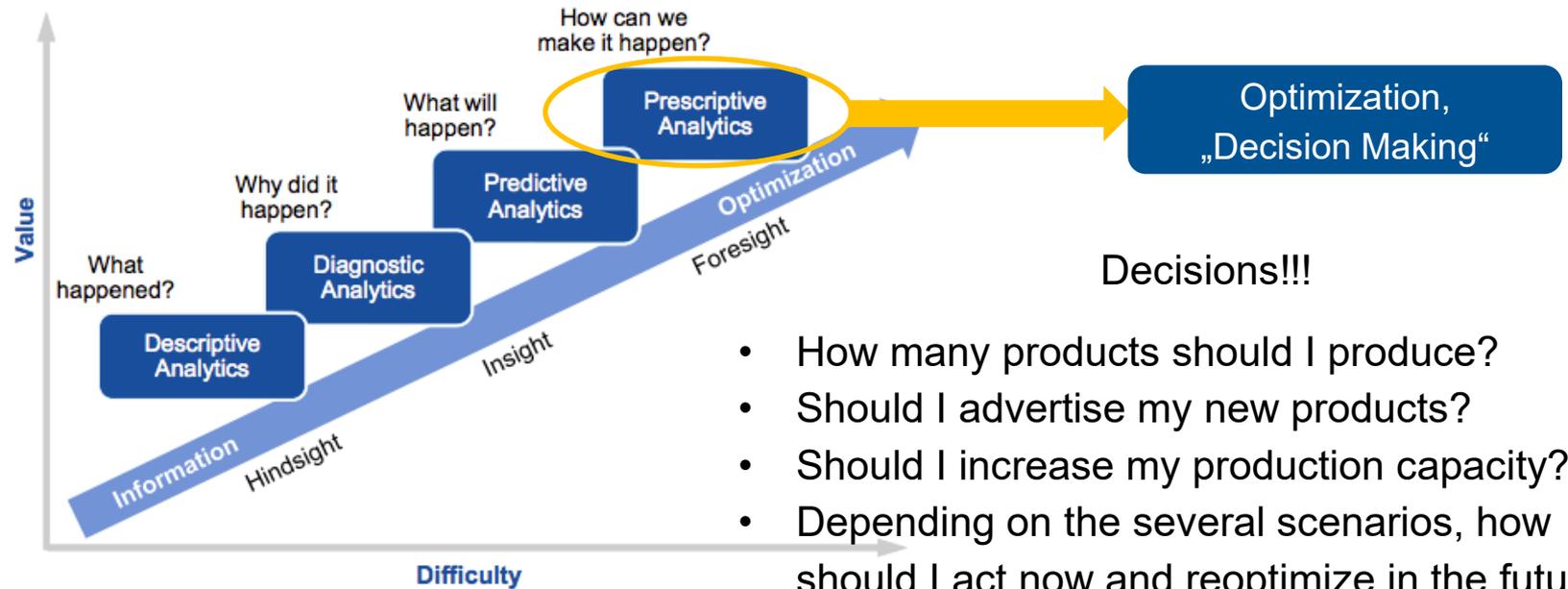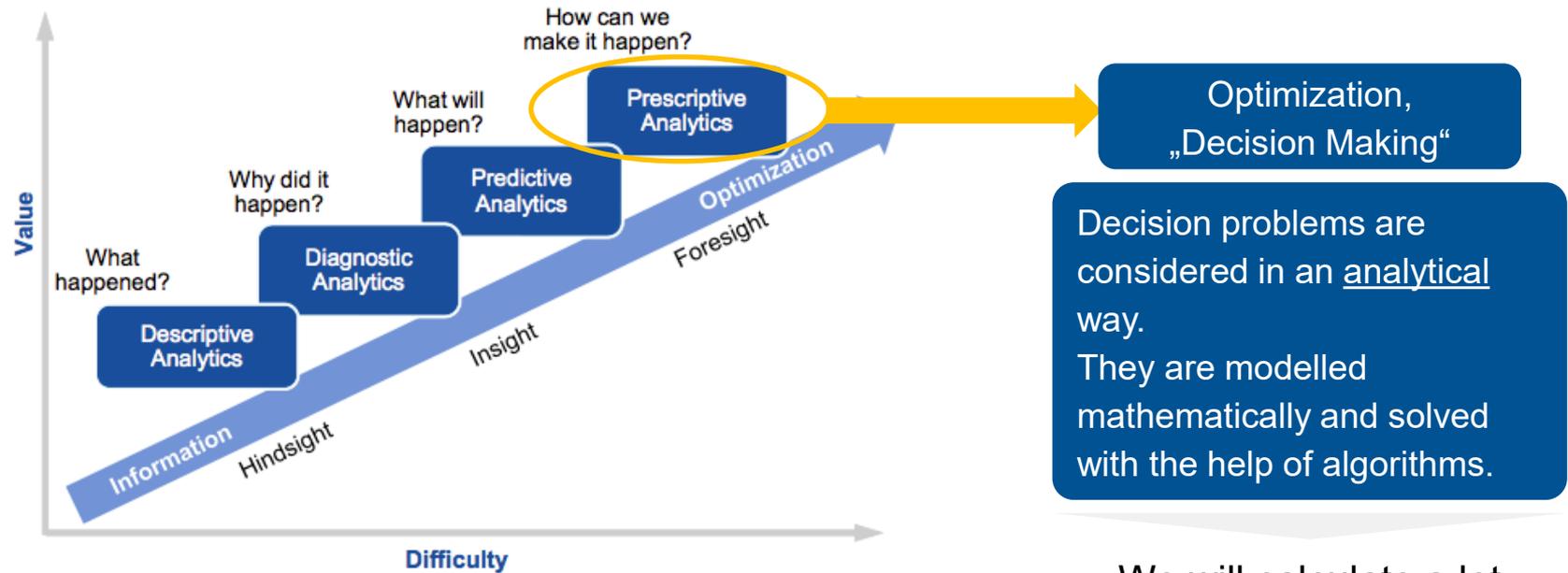
Decision problems are considered in an _analytical_ way.
They are modelled mathematically and solved with the help of algorithms.
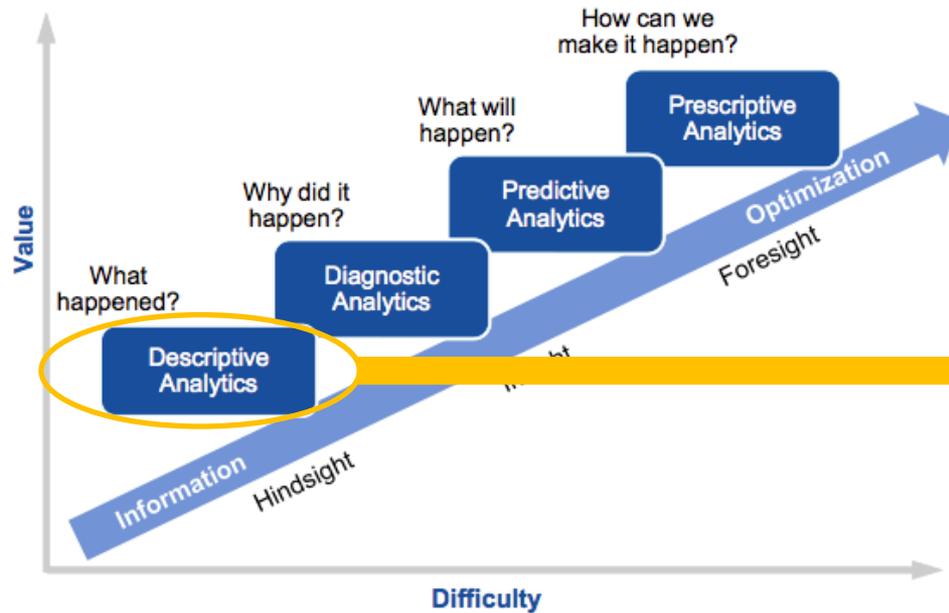
We will calculate a lot

What is Operations Research?
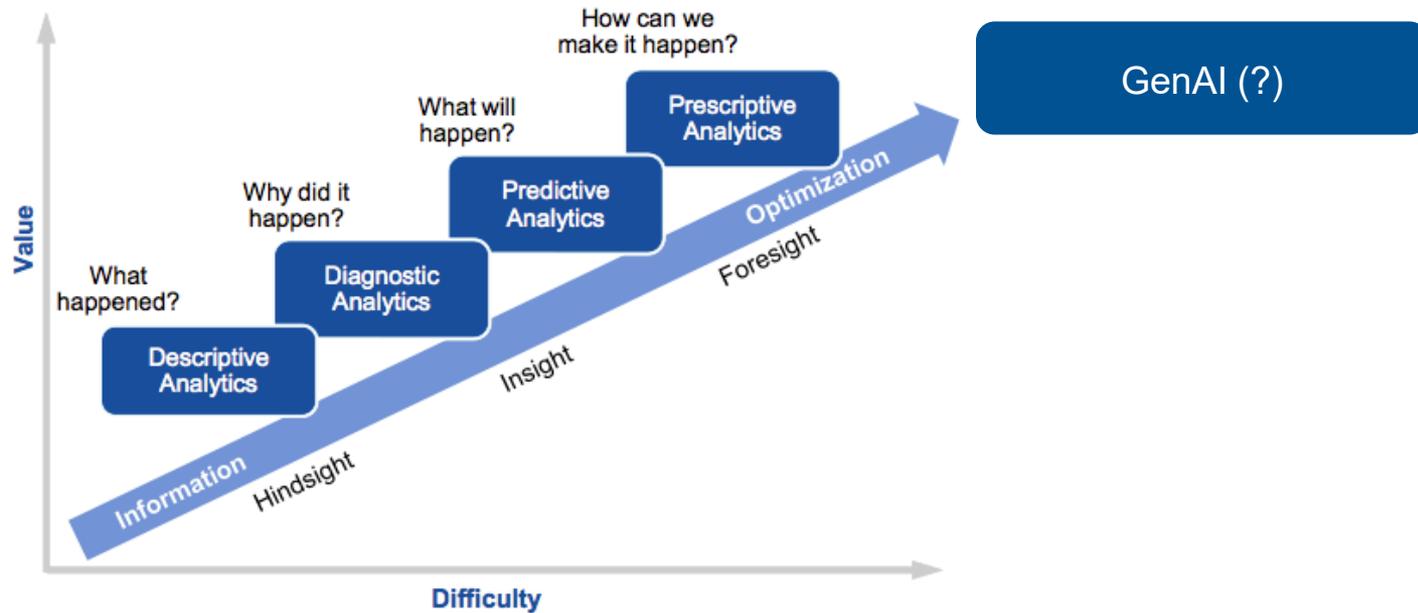
Source: Gartner (March 2012)

What is Operations Research?

In which industries are these solution techniques applicable?

In industry:

- Manufacturing
- Telecommunication
- Finance
- Process Industry
- Inventory Control
- Transportation
- Game Theory
- …

Examples from our chair:

- Sequencing and scheduling of aircraft landings and departures
- Planning of charging infrastructures for electric vehicles
- Optimize shift plans with respect to legal requirements
- Reduction of waiting times in healthcare operations
- Routing problems with respect to driver preferences

What is Operations Research?

Chapter 1: Decision Analysis

→ How to make decisions under uncertainty?

→ How to optimize for multiple objectives?

→ How to adjust for varying preference of decision makers?

Chapter 2: Linear Programming

→ How can we model and solve linear optimization problems?
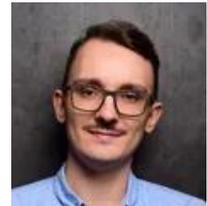
Chapter 3: Integer Programming

→ What if our decision variables are restricted to integer values?

What is Operations Research?

How do we tackle these decision making problems in this course?

Chapter 4: Graph Theory and Network Flow Problems
→ How can we efficiently solve network problems?

Chapter 5: Dynamic Programming
→ How do we model and solve sequential decision-making problems?
→ How do we make use of certain problem properties to efficiently solve them?

Decision Theory – Lecture Recap

Various ways to distinguish between different decision situations:

| Level of Uncertainty |

Decision under Certainty
The future is known and described by a single scenario. („deterministic")

Decision under Uncertainty
There are multiple different scenarios for the future, however their probabilities are unknown.
Number of scenarios: $n > 1$

Decision under Risk
There are multiple different scenarios for the future, their probabilities are known.

Number of scenarios: $n > 1$
Each scenario $j = 1, \dots, n$ has a probability of $0 < p_j < 1$ with $\sum_{j=1}^{n} p_j = 1$

Decision Theory – Lecture Recap

Various ways to distinguish between different decision situations:

Number of goals

Single criterion decision	Multi-criteria decision
Single criterion decision (Single goal)	**Multi-criteria decision** (Multiple goals)

Decision Theory – Lecture Recap

Various ways to distinguish between different decision situations:

Number of decision makers

Single (one) decision maker

Group decisions
(Multiple decision maker)

Decision Theory – Lecture Recap

Various ways to distinguish between different decision situations:

Time distinction

Static	Dynamic
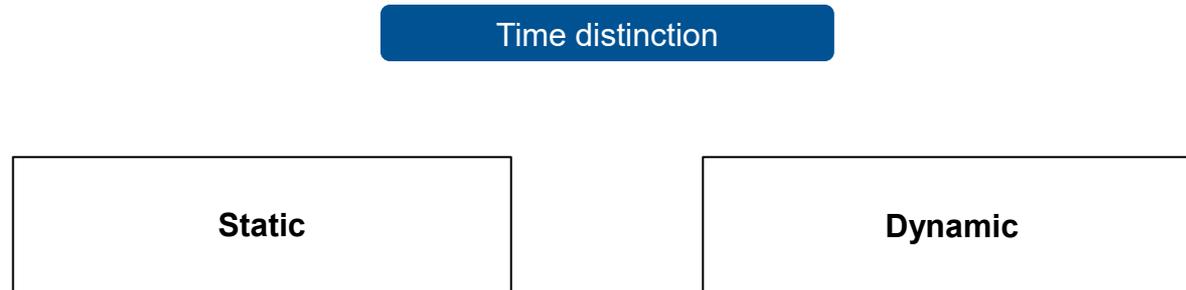
Decision Theory – Lecture Recap

Various ways to distinguish between different decision situations:

Behavior of opponent

Randomly acting opponent
(Game against nature)

Strategically acting opponent
(Game against active antagonist)

Decision Theory – Lecture Recap

Decision making under uncertainty:
- Uncertainty
- Single goal
- Single decision maker
- Static decision context
- Game against nature

→ There are m alternatives („actions") $a_1, \dots, a_m$ of which the decision maker has to take one

→ There are n different scenarios $s_1, \dots, s_n$ which may unfold and of which exactly one scenario will occur

Decision Theory – Lecture Recap

Decision making under uncertainty:

- Uncertainty
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→ There are m alternatives („actions") $a_1, \dots, a_m$ of which the decision maker has to take one

→ There are n different scenarios $s_1, \dots, s_n$ which may unfold and of which exactly one scenario will occur

Sequence of actions and events:

Action (a_i) → Scenario (s_j) → Outcome $(e_{i,j})$

Decision Theory – Lecture Recap

Decision making under uncertainty:
- Uncertainty
- Single goal
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- Static decision context
- Game against nature

→ There are m alternatives („actions") $a_1, ..., a_m$ of which the decision maker has to take one

→ There are n different scenarios $s_1, ..., s_n$ which may unfold and of which exactly one scenario will occur

Sequence of actions and events:

Action $(a_i) \rightarrow$ Scenario $(s_j) \rightarrow$ Outcome $(e_{i,j})$

→ Which action to take? What is the objective?
- Maximize the potential maximum gain?
- Minimize the maximum potential loss?
- Minimize the ‚regret'?

Different decision rules:

- Maximax-Rule
- Maximin-Rule
- Minimax-Regret-Rule
- Hurwicz-Rule
- Laplace-Rule

Decision Theory – Lecture Recap

<u>Depiction in a decision matrix:</u>

- Actions: $a_1, a_2, \ldots, a_i, \ldots, a_m$
- Scenarios: $s_1, s_2, \ldots, s_j, \ldots, s_n$
- Outcomes: $e_{1,1}, \ldots, e_{m,n}$

	s_1	..	s_j	..	s_n
a_1	$e_{1,1}$	..	$e_{1,j}$	..	$e_{1,n}$
..					
a_i	$e_{i,1}$	..	$e_{i,j}$	..	$e_{i,n}$
..					
a_m	$e_{m,1}$	..	$e_{m,j}$	..	$e_{m,n}$

Dominance criterion

An action a_i dominates another action a_j, if it generates an equal or better
outcome in every scenario $s_1, \ldots, s_n$

Efficient alternative

An action a_i is efficient, if it is not dominated by another alternative.

Decision Theory – Lecture Recap

Depiction in a decision matrix:

- Actions: $a_1, a_2, \ldots, a_i, \ldots, a_m$
- Scenarios: $s_1, s_2, \ldots, s_j, \ldots, s_n$
- Outcomes: $e_{1,1}, \ldots, e_{m,n}$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_3	3	-1	2
a_4	2	2	2

Let's assume a maximization problem.
4 alternatives
3 scenarios

Dominance criterion

An action a_i dominates another action a_j, if it generates an equal or better outcome in every scenario $s_1, \ldots, s_n$

Efficient alternative

An action a_i is efficient, if it is not dominated by another alternative.

Decision Theory – Lecture Recap

Depiction in a decision matrix:

- Actions: $a_1, a_2, \ldots, a_i, \ldots, a_m$
- Scenarios: $s_1, s_2, \ldots, s_j, \ldots, s_n$
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	s_1	s_2	s_3
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Let's assume a maximization problem.
4 alternatives
3 scenarios

Dominance criterion
An action a_i dominates another action a_j, if it generates an equal or better outcome in every scenario $s_1, \ldots, s_n$

Efficient alternative
An action a_i is efficient, if it is not dominated by another alternative.

Action a_3 is dominated by a_2.

Actions a_1, a_2 and a_4 are efficient.

Decision Theory – Lecture Recap

<u>No need to consider a_3:</u>

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_3	3	-1	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

Decision Theory – Lecture Recap

Maximin-Rule:

The decision maker expects the worst outcome.
The decision maker therefore wants to maximize the outcome in the worst-case scenario.

$$\max_i \{\min_j \{e_{i,j}\}\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\min_j\{e_{i,j}\}$
a_1	1	-1	3	-1
a_2	4	0	2	0
a_4	2	2	2	2

Decision Theory – Lecture Recap

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	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\min_{j}\{e_{i,j}\}$
a_1	1	-1	3	-1
a_2	4	0	2	0
a_4	2	2	2	2

$\max_{i}\{\min_{j}\{e_{i,j}\}\}$

Decision maker chooses alternative a_4

Decision Theory – Lecture Recap

Maximax-Rule:

The decision maker expects the best outcome.
The decision maker therefore wants to maximize the outcome in the best-case scenario.

$$\max_i \{\max_j \{e_{i,j}\}\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_1	1	-1	3	3
a_2	4	0	2	4
a_4	2	2	2	2

Decision Theory – Lecture Recap

Maximax-Rule:

The decision maker expects the best outcome.
The decision maker therefore wants to maximize the outcome in the best-case scenario.

$$\max_i \{\max_j\{e_{i,j}\}\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_1	1	-1	3	3
a_2	4	0	2	4
a_4	2	2	2	2

$\max_i \{\max_j\{e_{i,j}\}\}$

Decision maker chooses alternative a_2

Decision Theory – Lecture Recap

Minimax-Regret-Rule:

The decision maker wants to minimize his ‚regret' $r_{i,j}$.
He wants to perform as closely as possible to the optimal decision in a given situation.
„… if I had only taken action 1 instead of action 2 …"

$$\min_{i} \{\max_{j}\{r_{i,j}\}\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

Decision Theory – Lecture Recap

Minimax-Regret-Rule:

The decision maker wants to minimize his ‚regret' $r_{i,j}$.

He wants to perform as closely as possible to the optimal decision in a given situation.

„… if I had only taken action 1 instead of action 2 …"

→ Determine regret matrix with entries $r_{i,j}$ first.

$$\min_i \{\max_j \{r_{i,j}\}\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2
$\max_i \{e_{i,j}\}$	4	2	3

	s_1	s_2	s_3	$\max_j \{r_{i,j}\}$
a_1				
a_2				
a_4				

Decision Theory – Lecture Recap

Minimax-Regret-Rule:

The decision maker wants to minimize his ‚regret' $r_{i,j}$.

He wants to perform as closely as possible to the optimal decision in a given situation.

„… if I had only taken action 1 instead of action 2 …"

→ Determine $\boxed{\text{regret matrix}}$ with entries $r_{i,j}$ first.
$$\min_i \{\max_j \{r_{i,j}\}\}$$

$$4 - 1 = 3$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2
$\max_i \{e_{i,j}\}$	4	2	3

	s_1	s_2	s_3	$\max_j \{r_{i,j}\}$
a_1	3			
a_2				
a_4				

Decision Theory – Lecture Recap

Minimax-Regret-Rule:

The decision maker wants to minimize his ‚regret' $r_{i,j}$.

He wants to perform as closely as possible to the optimal decision in a given situation.

„… if I had only taken action 1 instead of action 2 …"

→ Determine regret matrix with entries $r_{i,j}$ first.

$$\min_i \{\max_j\{r_{i,j}\}\}$$

$$2 - (-1) = 3$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2
$\max_i\{e_{i,j}\}$	4	2	3

	s_1	s_2	s_3	$\max_j\{r_{i,j}\}$
a_1	3	3		
a_2				
a_4				

Decision Theory – Lecture Recap

Minimax-Regret-Rule:

The decision maker wants to minimize his ‚regret' $r_{i,j}$.

He wants to perform as closely as possible to the optimal decision in a given situation.

„… if I had only taken action 1 instead of action 2 …"

→ Determine ⟦regret matrix⟧ with entries $r_{i,j}$ first. $\min_i \{\max_j \{r_{i,j}\}\}$ $4 - 4 = 0$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2
$\max_i \{e_{i,j}\}$	4	2	3

	s_1	s_2	s_3	$\max_j \{r_{i,j}\}$
a_1	3	3		
a_2	0			
a_4				

Decision Theory – Lecture Recap

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The decision maker wants to minimize his ‚regret' $r_{i,j}$.

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„… if I had only taken action 1 instead of action 2 …"

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	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2
$\max_i\{e_{i,j}\}$	4	2	3

	s_1	s_2	s_3	$\max_j\{r_{i,j}\}$
a_1	3	3	0	3
a_2	0	2	1	2
a_4	2	0	1	2

Decision Theory – Lecture Recap

Minimax-Regret-Rule:

The decision maker wants to minimize his ‚regret' $r_{i,j}$.

He wants to perform as closely as possible to the optimal decision in a given situation.

„… if I had only taken action 1 instead of action 2 …"

→ Determine regret matrix with entries $r_{i,j}$ first.　$\boxed{\min_{i}} \{\max_{j}\{r_{i,j}\}\}$

Decision maker chooses
alternative a_2 or alternative a_4

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2
$\max_{i}\{e_{i,j}\}$	4	2	3

	s_1	s_2	s_3	$\max_{j}\{r_{i,j}\}$
a_1	3	3	0	3
a_2	0	2	1	2
a_4	2	0	1	2

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j\{e_{i,j}\} + (1 - \lambda) * \min_j\{e_{i,j}\} \right\}$$

Maximax   Maximin

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

$\rightarrow$ Assume $\lambda = 0.3$

	s_1	s_2	s_3			s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$
a_1	1	-1	3		a_1	1	-1	3		
a_2	4	0	2		a_2	4	0	2		
a_4	2	2	2		a_4	2	2	2		

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1-\lambda) * \min_j \{e_{i,j}\} \right\}$$

→ Assume $\lambda = 0.3$

	s_1	s_2	s_3			s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$
a_1	1	-1	3		a_1	1	-1	3	3	
a_2	4	0	2		a_2	4	0	2	4	
a_4	2	2	2		a_4	2	2	2	2	

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

→ Assume $\lambda = 0.3$

	s_1	s_2	s_3			s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$
a_1	1	-1	3		a_1	1	-1	3	3	-1
a_2	4	0	2		a_2	4	0	2	4	0
a_4	2	2	2		a_4	2	2	2	2	2

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

→ Assume $\lambda = 0.3$

$$0.3 * 3 + (1 - 0.3) * (-1) = 0.2$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$	e_i
a_1	1	-1	3	3	-1	0.2
a_2	4	0	2	4	0	
a_4	2	2	2	2	2	

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

$\rightarrow$ Assume $\lambda = 0.3$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$	e_i
a_1	1	-1	3	3	-1	0.2
a_2	4	0	2	4	0	1.2
a_4	2	2	2	2	2	2

Decision Theory – Lecture Recap

Hurwicz-Rule:

Choose the appropriate action where the linear combination of minimal and maximal outcome is maximized. The weight is given by optimism-parameter λ.

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

$\rightarrow$ Assume $\lambda = 0.3$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$	e_i
a_1	1	-1	3	3	-1	0.2
a_2	4	0	2	4	0	1.2
a_4	2	2	2	2	2	2

Decision maker chooses alternative a_4

Decision Theory – Lecture Recap

Hurwicz-Rule:

→ Assume $\lambda = 0.3$

$$\max_i \left\{ \lambda * \max_j\{e_{i,j}\} + (1-\lambda) * \min_j\{e_{i,j}\} \right\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$	e_i
a_1	1	-1	3	3	-1	0.2
a_2	4	0	2	4	0	1.2
a_4	2	2	2	2	2	2

Decision maker chooses alternative a_4

Decision Theory – Lecture Recap

Hurwicz-Rule:

→ Assume $\lambda = 0.3$

$$\max_i \left\{ \lambda * \max_j \{e_{i,j}\} + (1 - \lambda) * \min_j \{e_{i,j}\} \right\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$	$\min_j\{e_{i,j}\}$	e_i
a_1	1	-1	3	3	-1	0.2
a_2	4	0	2	4	0	1.2
a_4	2	2	2	2	2	2

Decision maker chooses alternative a_4

Decision Theory – Lecture Recap

Laplace-Rule:

Choose the appropriate action having the highest expected value under the assumption that all scenarios have the same probability.

$$\max_i \left\{ \frac{1}{n} \sum_{k=1}^{n} e_{i,k} \right\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	e_i
a_1	1	-1	3	
a_2	4	0	2	
a_4	2	2	2	

Decision Theory – Lecture Recap

Laplace-Rule:

Choose the appropriate action having the highest expected value under the assumption that all scenarios have the same probability.

$$\max_i \left\{ \frac{1}{n} \sum_{k=1}^{n} e_{i,k} \right\}$$

$$\frac{1}{3} * (1 + (-1) + 3) = 1$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	e_i
a_1	1	-1	3	1
a_2	4	0	2	
a_4	2	2	2	

Decision Theory – Lecture Recap

Laplace-Rule:

Choose the appropriate action having the highest expected value under the assumption that all scenarios have the same probability.

$$\max_i \left\{ \frac{1}{n} \sum_{k=1}^{n} e_{i,k} \right\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	e_i
a_1	1	-1	3	1
a_2	4	0	2	2
a_4	2	2	2	2

Decision Theory – Lecture Recap

Laplace-Rule:

Choose the appropriate action having the highest expected value under the assumption that all scenarios have the same probability.

$$\max_i \left\{ \frac{1}{n} \sum_{k=1}^{n} e_{i,k} \right\}$$

	s_1	s_2	s_3
a_1	1	-1	3
a_2	4	0	2
a_4	2	2	2

	s_1	s_2	s_3	e_i
a_1	1	-1	3	1
a_2	4	0	2	2
a_4	2	2	2	2

Decision maker chooses alternative a_2 or alternative a_4

Exercise: Question 3

Question 3

Consider the following decision matrix for a maximization problem:

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

Determine the integer value(s) for $x \in \mathbb{N}$ such that all alternatives are efficient or non-dominated.

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?
  → will not dominate the other one, independent of x, see e.g. s_1, s_2

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?
  → will not dominate the other one, independent of x, see e.g. s_1, s_2

- a_1 and a_4?

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?
  → will not dominate the other one, independent of x, see e.g. s_1, s_2

- a_1 and a_4?
  → $x > 10$

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?
  → will not dominate the other one, independent of x, see e.g. s_1, s_2

- a_1 and a_4?
  → $x > 10$

- a_3 and a_4?

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?
  → will not dominate the other one, independent of x, see e.g. s_1, s_2

- a_1 and a_4?
  → $x > 10$

- a_3 and a_4?
  → $x < 13$

Exercise: Question 3

	s_1	s_2	s_3	s_4	s_5
a_1	13	21	25	18	10
a_2	14	15	20	14	18
a_3	11	18	24	13	13
a_4	12	19	25	13	x

- Neither a_1 nor a_2 dominate the other one respectively
- Neither a_1 nor a_3 dominate the other one respectively
- Neither a_2 nor a_3 dominate the other one respectively
  → Only task left: compare a_1, a_2 and a_3 to a_4

- Let's start with a_2 and a_4: Dominance depending on x?
  → will not dominate the other one, independent of x, see e.g. s_1, s_2

- a_1 and a_4?
  → $x > 10$

- a_3 and a_4?
  → $x < 13$

All alternatives are efficient if $x \in \{11,12\}$

Exercise: Winter Term 2014 / 2015 Q 4.1

4.1 (4P) Consider the following decision matrix for a <u>minimization</u> problem:

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	-1	3	3

a) Determine the efficient actions. Motivate your answer.   (1P)

b) Use the Laplace rule to select an action.   (1P)

c) Use the Minimax Regret rule to select an action.   (2P)

Exercise: Winter Term 2014 / 2015 Q 4.1

a) Efficient alternatives?

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	-1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>a) Efficient alternatives?</u>

- a_1 outperforms a_3 in scenario s_1.
  In every other scenario, a_1 does not
  perform worse than a_3.
  Therefore a_1 dominates a_3

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	-1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>a) Efficient alternatives?</u>

- a_1 outperforms a_3 in scenario s_1.
  In every other scenario, a_1 does not
  perform worse than a_3.
  Therefore a_1 dominates a_3

- a_1 outperforms a_4 in scenario s_2.
  In every other scenario, a_1 does not
  perform worse than a_4.
  Therefore a_1 dominates a_4

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	-1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

a) Efficient alternatives?

- a_1 outperforms a_3 in scenario s_1.
  In every other scenario, a_1 does not
  perform worse than a_3.
  Therefore a_1 dominates a_3

- a_1 outperforms a_4 in scenario s_2.
  In every other scenario, a_1 does not
  perform worse than a_4.
  Therefore a_1 dominates a_4

- a_1 performs better in s_1 than a_2.
  a_2 performs better in s_2 than a_1.
  Therefore no dominance.

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	-1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>a) Efficient alternatives?</u>

- a_1 outperforms a_3 in scenario s_1.
  In every other scenario, a_1 does not
  perform worse than a_3.
  Therefore a_1 dominates a_3

- a_1 outperforms a_4 in scenario s_2.
  In every other scenario, a_1 does not
  perform worse than a_4.
  Therefore a_1 dominates a_4

- a_1 performs better in s_1 than a_2.
  a_2 performs better in s_2 than a_1.
  Therefore no dominance.

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

Efficient alternatives: a_1, a_2

Exercise: Winter Term 2014 / 2015 Q 4.1

b) Use the Laplace-Rule to select an action.

- For alternative a_1:

$$\text{Laplace: } \sum_{j=1}^{n} e_{i,j} \xrightarrow{n=3} \frac{1}{3}(-1+2+3)$$

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

- For alternative a_2:

Exercise: Winter Term 2014 / 2015 Q 4.1

b) Use the Laplace-Rule to select an action.

- For alternative a_1:

$$\frac{1}{3} * (-1 + 2 + 3) = \frac{4}{3}$$

- For alternative a_2:

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

b) Use the Laplace-Rule to select an action.

- For alternative a_1:

$$\frac{1}{3} * (-1 + 2 + 3) = \frac{4}{3}$$

- For alternative a_2:

$$\frac{1}{3} * (1 + 1 + 3) = \frac{5}{3}$$

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>b) Use the Laplace-Rule to select an action.</u>

- For alternative a_1:

$$\frac{1}{3} * (-1 + 2 + 3) = \frac{4}{3}$$

Select alternative …?

- For alternative a_2:

$$\frac{1}{3} * (1 + 1 + 3) = \frac{5}{3}$$

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>b) Use the Laplace-Rule to select an action.</u>

- For alternative a_1:

$$\frac{1}{3} * (-1 + 2 + 3) = \frac{4}{3}$$

- For alternative a_2:

$$\frac{1}{3} * (1 + 1 + 3) = \frac{5}{3}$$

Select alternative a_1.
(Remember: Minimization-Problem)

4.1 (4P) Consider the following decision matrix for a <u>minimization</u> problem:

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

c) Use the Minimax-Regret-Rule to select an action.

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
a_3	0	2	3
a_4	1	3	3

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>c) Use the Minimax-Regret-Rule to select an action.</u>

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
~~a_3~~	~~0~~	~~2~~	~~3~~
~~a_4~~	~~1~~	~~3~~	~~3~~

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
$\min_i\{e_{i,j}\}$	-1	1	3

„best" possible outcome for each scenario

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>c) Use the Minimax-Regret-Rule to select an action.</u>

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
~~a_3~~	~~0~~	~~2~~	~~3~~
~~a_4~~	~~1~~	~~3~~	~~3~~

Regret-Matrix:

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
$\min_i\{e_{i,j}\}$	-1	1	3

➤

	s_1	s_2	s_3	$\max_j\{r_{i,j}\}$
a_1	0	1	0	1
a_2	2	0	0	2

Regret: Difference to best possible outcome of each scenario

Exercise: Winter Term 2014 / 2015 Q 4.1

<u>c) Use the Minimax-Regret-Rule to select an action.</u>

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
~~a_3~~	~~0~~	~~2~~	~~3~~
~~a_4~~	~~1~~	~~3~~	~~3~~

Regret-Matrix:

	s_1	s_2	s_3
a_1	-1	2	3
a_2	1	1	3
$\min_{i}\{e_{i,j}\}$	-1	1	3

	s_1	s_2	s_3	$\max_{j}\{r_{i,j}\}$
a_1	0	1	0	1
a_2	2	0	0	2

Choose alternative a_1

4.1 (4P) Consider the following decision matrix for a <u>min</u>imization problem:

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

a) Determine the efficient alternatives depending on x with x in the interval $[-10, -9, \ldots, 9, 10]$. Provide an argument for your answer. (2P)

b) Determine the maximum value for x for which a decision maker applying the Laplace rule would choose alternative a_1. Provide a rationale for your answer. (2P)

Exercise: Summer Term 2016 Q 4.1

a) Efficient alternatives depending on x with $x \in [-10, ..., 10]$

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

Exercise: Summer Term 2016 Q 4.1

<u>a) Efficient alternatives depending on x with $x \in [-10, ..., 10]$</u>

Which values of x are of interest?

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

Exercise: Summer Term 2016 Q 4.1

<u>a) Efficient alternatives depending on x with $x \in [-10, \dots, 10]$</u>

Which values of x are of interest?

- When $x = 2$ → a_1 and a_2 are equal
  When $x < 2$ → a_1 dominates a_2
  When $x > 2$ → a_2 dominates a_1

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

Exercise: Summer Term 2016 Q 4.1

a) Efficient alternatives depending on x with $x \in [-10, ..., 10]$

Which values of x are of interest?

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

- When $x = 2$ → a_1 and a_2 are equal
  When $x < 2$ → a_1 dominates a_2
  When $x > 2$ → a_2 dominates a_1

- When $x < -2$ → no dominance between a_1 and a_3
  Even though a_1 outperforms a_3 in s_1, a_3 outperforms
  a_1 in s_2. Not of relevance.

Exercise: Summer Term 2016 Q 4.1

<u>a) Efficient alternatives depending on x with $x \in [-10, \dots, 10]$</u>

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

Which values of x are of interest?

- When $x = 2$ → a_1 and a_2 are equal
  When $x < 2$ → a_1 dominates a_2
  When $x > 2$ → a_2 dominates a_1

- When $x < -2$ → no dominance between a_1 and a_3
  Even though a_1 outperforms a_3 in s_1, a_3 outperforms a_1 in s_2. Not of relevance.

- When $x \geq 5$ → a_3 dominates a_1 (redundant because a_1 is already dominated by a_2 when $x > 2$.)

Exercise: Summer Term 2016 Q 4.1

<u>a) Efficient alternatives depending on x with $x \in [-10, ..., 10]$</u>

Which values of x are of interest?

- When $x = 2$ → a_1 and a_2 are equal
  When $x < 2$ → a_1 dominates a_2
  When $x > 2$ → a_2 dominates a_1

- When $x < -2$ → no dominance between a_1 and a_3
  Even though a_1 outperforms a_3 in s_1, a_3 outperforms a_1 in s_2. Not of relevance.

- When $x \geq 5$ → a_3 dominates a_1 (redundant because a_1 is already dominated by a_2 when $x > 2$.)

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

x	Efficient alternatives
$-10 \leq x < 2$	a_1, a_3
$x = 2$	a_1, a_2, a_3
$2 < x \leq 10$	a_2, a_3

b) Determine the largest value of x so that a decision maker chooses alternative a_1 when using the Laplace rule.

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

The following conditions have to hold:

a_1 better than a_2:

a_1 better than a_3:

b) Determine the largest value of x so that a decision maker chooses alternative a_1 when using the Laplace rule.

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

The following conditions have to hold:

a_1 better than a_2:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (2 + 1 + 2)$$

a_1 better than a_3:

Exercise: Summer Term 2016 Q 4.1

b) Determine the largest value of x so that a decision maker chooses alternative a_1 when using the Laplace rule.

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

The following conditions have to hold:

a_1 better than a_2:

$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (2 + 1 + 2) \quad \rightarrow \quad x < 2$

a_1 better than a_3:

Exercise: Summer Term 2016 Q 4.1

b) Determine the largest value of x so that a decision maker chooses alternative a_1 when using the Laplace rule.

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

The following conditions have to hold:

a_1 better than a_2:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (2 + 1 + 2) \quad \rightarrow \quad x < 2$$

a_1 better than a_3:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (5 - 1 - 2)$$

Exercise: Summer Term 2016 Q 4.1

b) Determine the largest value of x so that a decision maker chooses alternative a_1 when using the Laplace rule.

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

The following conditions have to hold:

a_1 better than a_2:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (2 + 1 + 2) \quad \rightarrow \quad x < 2$$

a_1 better than a_3:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (5 - 1 - 2) \quad \rightarrow \quad x < \frac{1}{2}$$

Exercise: Summer Term 2016 Q 4.1

b) Determine the largest value of x so that a decision maker chooses alternative a_1 when using the Laplace rule.

	s_1	s_2	s_3
a_1	x	1	x
a_2	2	1	2
a_3	5	-1	-2

The following conditions have to hold:

a_1 better than a_2:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (2 + 1 + 2) \quad \rightarrow \quad x < 2$$

a_1 better than a_3:

$$\frac{1}{3} * (x + 1 + x) < \frac{1}{3} * (5 - 1 - 2) \quad \rightarrow \quad x < \frac{1}{2}$$

$$x < \frac{1}{2}$$

Exercise: Question 8

Question 8

A tour operator has the chance to buy 0, 1000, 2000 or 3000 nights for 90 Euro per night in a hotel in Antalya for the next season. If he sells a night within one of his package holidays, he would earn 100 Euro for that night. He estimates to be able to sell 1800 nights in an average year and 900 (3000) nights in a bad (good) year.

a) Construct a decision matrix.

b) Remove all dominated alternatives, if any, and reconstruct the decision matrix to include only the efficient alternatives.

c) How many nights should the tour operator buy, if he decides under one of the following rules?

   (1) Maximax rule

   (2) Maximin rule

   (3) Laplace rule

   (4) Minimax Regret rule

Exercise: Question 8

Actions:

„…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Exercise: Question 8

Actions:

„…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Scenarios:

„… good year, average year, bad year …"

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

Exercise: Question 8

<u>Actions:</u>

„…to buy 0, 1000, 2000 or 3000 nights …"

a_1: 0

a_2: 1000

a_3: 2000

a_4: 3000

<u>Scenarios:</u>

„… good year, average year, bad year …"

s_1: 3000

s_2: 1800

s_3: 900

<u>Also:</u>

Cost per night: 90 EUR

Revenue per night: 100 EUR

> Tour operator wants to <u>maximize</u> profits.

Exercise: Question 8

Actions:

„…to buy 0, 1000, 2000 or 3000 nights …"

a_1: 0

a_2: 1000

a_3: 2000

a_4: 3000

Scenarios:

„… good year, average year, bad year …"

s_1: 3000

s_2: 1800

s_3: 900

Also:

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3			
a_4			

Exercise: Question 8

Actions:

"…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Scenarios:

"… good year, average year, bad year …"

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

Also:

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3	20000		
a_4			

$$Cost: 2000\ Nights * 90\frac{EUR}{Night} = 180000$$

$$Revenue: 2000\ Nights * 100\frac{EUR}{Night} = 200000$$

$$Profit: 200000 - 180000 = 20000$$

Exercise: Question 8

Actions:

"…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Scenarios:

"… good year, average year, bad year …"

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

Also:

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3	20000	0	
a_4			

$$Cost: 2000\ Nights * 90\ \frac{EUR}{Night} = 180000$$

$$Revenue: 1800\ Nights * 100\ \frac{EUR}{Night} = 180000$$

$$Profit: 180000 - 180000 = 0$$

Exercise: Question 8

Actions:

„…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Scenarios:

„… good year, average year, bad year …"

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

Also:

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3	20000	0	-90000
a_4			

$$Cost: 2000 \; Nights * 90 \frac{EUR}{Night} = 180000$$

$$Revenue: 900 \; Nights * 100 \frac{EUR}{Night} = 90000$$

$$Profit: 90000 - 180000 = -90000$$

Exercise: Question 8

Actions:

„…to buy 0, 1000, 2000 or 3000 nights …“

a_1: 0

a_2: 1000

a_3: 2000

a_4: 3000

Scenarios:

„… good year, average year, bad year …“

s_1: 3000

s_2: 1800

s_3: 900

Also:

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1	0	0	0
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

<u>Actions:</u>

„…to buy 0, 1000, 2000 or 3000 nights …"

a_1: 0

a_2: 1000

a_3: 2000

a_4: 3000

<u>Scenarios:</u>

„… good year, average year, bad year …"

s_1: 3000

s_2: 1800

s_3: 900

<u>Also:</u>

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1	0	0	0
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Efficient alternatives?

Exercise: Question 8

Actions:

„…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Scenarios:

„… good year, average year, bad year …"

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

Also:

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1	0	0	0
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Efficient alternatives?

Exercise: Question 8

<u>Decision based on Maximax-Rule:</u>

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

<u>Decision based on Maximax-Rule:</u>

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_2	10000	10000	0	10000
a_3	20000	0	-90000	20000
a_4	30000	-90000	-180000	30000

Exercise: Question 8

Decision based on Maximax-Rule:

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_2	10000	10000	0	10000
a_3	20000	0	-90000	20000
a_4	30000	-90000	-180000	30000

▶ Choose alternative a_4

Exercise: Question 8

Decision based on Maximin-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

<u>Decision based on Maximin-Rule:</u>

	s_1	s_2	s_3	$\min_j\{e_{i,j}\}$
a_2	10000	10000	0	0
a_3	20000	0	-90000	-90000
a_4	30000	-90000	-180000	-180000

▶ Choose alternative a_2

Exercise: Question 8

<u>Decision based on Laplace-Rule:</u>

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

<u>Decision based on Laplace-Rule:</u>

	s_1	s_2	s_3	$\frac{1}{3}\sum_{j=1}^{3} e_{i,j}$
a_2	10000	10000	0	6666.66
a_3	20000	0	-90000	-23333.33
a_4	30000	-90000	-180000	-80000

Exercise: Question 8

Decision based on Laplace-Rule:

	s_1	s_2	s_3	$\dfrac{1}{3}\displaystyle\sum_{j=1}^{3} e_{i,j}$
a_2	10000	10000	0	6666.66
a_3	20000	0	-90000	-23333.33
a_4	30000	-90000	-180000	-80000

▶ Choose alternative a_2

Exercise: Question 8

<u>Decision based on Minimax-Regret-Rule:</u>

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

<u>Decision based on Minimax-Regret-Rule:</u>

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000
$\max_i\{e_{i,j}\}$	30000	10000	0

Exercise: Question 8

Decision based on Minimax-Regret-Rule:

Regret-Matrix:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000
$\max_i\{e_{i,j}\}$	30000	10000	0

	s_1	s_2	s_3
a_2	20000	0	0
a_3	10000	10000	90000
a_4	0	100000	180000

Exercise: Question 8

Decision based on Minimax-Regret-Rule:

Regret-Matrix:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000
$\max_i\{e_{i,j}\}$	30000	10000	0

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_2	20000	0	0	20000
a_3	10000	10000	90000	90000
a_4	0	100000	180000	180000

Choose alternative a_2

Decision under Risk: Definitions & Notation

Various ways to distinguish between different decision situations:

Level of Uncertainty

Decision under Certainty
The future is known and described by a single scenario. („deterministic")

Decision under Uncertainty
There are multiple different scenarios for the future, however their probabilities are unknown.
Number of scenarios: $n > 1$

Decision under Risk
There are multiple different scenarios for the future, their probabilities are known.

Number of scenarios: $n > 1$
Each scenario $j = 1, ..., n$ has a probability of $0 < p_j < 1$
with $\sum_{j=1}^{n} p_j = 1$

Decision under Risk: Definitions & Notation

Decision making under risk:

- Risk
- One criterion
- Single decision maker
- Static decision

→ There are m alternatives („actions") $a_1, \ldots, a_m$ of which the decision maker has to take one

→ These actions can also be regarded as lotteries $l_1, \ldots, l_m$

→ There are n different scenarios $s_1, \ldots, s_n$ which may unfold and of which exactly one scenario will occur

→ The probability that a scenario s_j occurs is known and denoted as p_j

Decision under Risk: Definitions & Notation

Extension of decision matrix with probabilities p_j:

	s_1	s_2	s_3
a_1	$e_{1,1}$	$e_{1,2}$	$e_{1,3}$
a_2	$e_{2,1}$	$e_{2,2}$	$e_{2,3}$
a_3	$e_{3,1}$	$e_{3,2}$	$e_{3,3}$
a_4	$e_{4,1}$	$e_{4,2}$	$e_{4,3}$

	p_1	p_2	p_3
	s_1	s_2	s_3
a_1	$e_{1,1}$	$e_{1,2}$	$e_{1,3}$
a_2	$e_{2,1}$	$e_{2,2}$	$e_{2,3}$
a_3	$e_{3,1}$	$e_{3,2}$	$e_{3,3}$
a_4	$e_{4,1}$	$e_{4,2}$	$e_{4,3}$

It holds: $\displaystyle\sum_{j=1}^{n} p_j = 1$ $\qquad 0 < p_j < 1$

Each row of the decision matrix under risk can be represented as a **lottery** $L_i = \left(p_1, e_{i,1}; \ldots; p_n, e_{i,n}\right)$

The **Expected Value (EV)** of a lottery is: $\displaystyle EV(L_i) = \sum_{j=1}^{n} p_j * e_{i,j}$

Decision under Risk: Definitions & Notation

Introduction of concept of (expected) utility:

- Not all decision makers behave rationally

- Different attitudes towards risk result in the selection of different lotteries

- Compare thought experiment St. Petersburg-Paradox:

  → Even though the Expected Value of the lottery is infinite,
     most people would exchange the lottery with a certain payout

  → The Expected Value is not always a suited method to value lotteries

  → The value of an outcome e is determined by the decision maker's
     personal risk attitude (captured in a utility function $u(x)$)

Nicolas
Bernoulli

Daniel
Bernoulli

Pictures' Source: Wikipedia

Thank you!

Baturhan Bayraktar