Operations Research and Decision Analysis
Exercise Session 2

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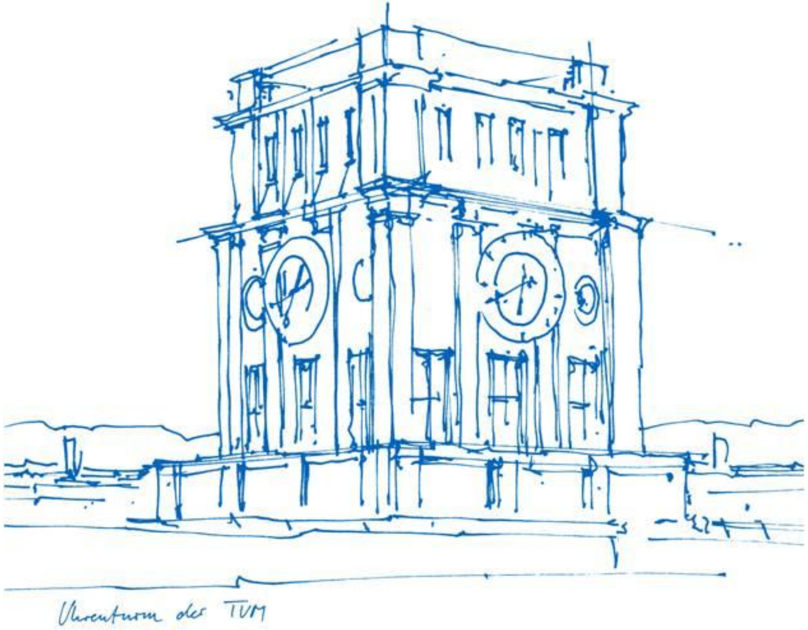


Agenda

1 Exercise: Question 8

2 Decision under Risk: Definitions & Notation

3 Exercise: Question 16

4 Exercise: Question 21

5 Exercise: Winter Term 2014 / 2015 Q4.2

6 Exercise: Question 27

Exercise: Question 8

Question 8
A tour operator has the chance to buy 0, 1000, 2000 or 3000 nights for 90 Euro per night in a hotel in Antalya for the next season. If he sells a night within one of his package holidays, he would earn 100 Euro for that night. He estimates to be able to sell 1800 nights in an average year and 900 (3000) nights in a bad (good) year.

a) Construct a decision matrix.

	s_1	s_2	s_3
a_1	0	0	0
a_2	0	10K	10K
a_3	-90K	0	20K
a_4	-180K	-90K	30K

P/L for each

b) Remove all dominated alternatives, if any, and reconstruct the decision matrix to include only the efficient alternatives. a_1 is dominated

c) How many nights should the tour operator buy, if he decides under one of the following rules?

(1) Maximax rule → a_4

(2) Maximin rule → a_2

(3) Laplace rule → a_2

(4) Minimax Regret rule

	s_1	s_2	s_3
a_2	/	/	20K
a_3	90K	10K	10K
a_4	180K	100K	/

→ a_2

Exercise: Question 8

<u>Actions:</u>

„…to buy 0, 1000, 2000 or 3000 nights …“

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

Exercise: Question 8

<u>Actions:</u>

„…to buy 0, 1000, 2000 or 3000 nights …“

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

<u>Scenarios:</u>

„… good year, average year, bad year …“

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

Exercise: Question 8

Actions:
„…to buy 0, 1000, 2000 or 3000 nights …“
$a_1: 0$
$a_2: 1000$
$a_3: 2000$
$a_4: 3000$

Scenarios:
„… good year, average year, bad year …“
$s_1: 3000$
$s_2: 1800$
$s_3: 900$

Also:
Cost per night: 90 EUR
Revenue per night: 100 EUR

Tour operator wants to maximize profits.

Exercise: Question 8

Actions:
„…to buy 0, 1000, 2000 or 3000 nights …“
$a_1: 0$
$a_2: 1000$
$a_3: 2000$
$a_4: 3000$

Scenarios:
„… good year, average year, bad year …“
$s_1: 3000$
$s_2: 1800$
$s_3: 900$

Also:
Cost per night: 90 EUR
Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3			
a_4			

Exercise: Question 8

<u>Actions:</u>
„…to buy 0, 1000, 2000 or 3000 nights …“
$a_1: 0$
$a_2: 1000$
$a_3: 2000$
$a_4: 3000$

<u>Scenarios:</u>
„… good year, average year, bad year …“
$s_1: 3000$
$s_2: 1800$
$s_3: 900$

<u>Also:</u>
Cost per night: 90 EUR
Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3	20000		
a_4			

$$Cost: 2000\ Nights * 90 \frac{EUR}{Night} = 180000$$

$$Revenue: 2000\ Nights\ * 100 \frac{EUR}{Night} = 200000$$

$$Profit: 200000 - 180000 = 20000$$

Exercise: Question 8

<u>Actions:</u>

„…to buy 0, 1000, 2000 or 3000 nights …“

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

<u>Scenarios:</u>

„… good year, average year, bad year …“

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

<u>Also:</u>

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3	20000	0	
a_4			

$$Cost: 2000\ Nights * 90 \frac{EUR}{Night} = 180000$$

$$Revenue: 1800\ Nights\ * 100 \frac{EUR}{Night} = 180000$$

$$Profit: 180000 - 180000 = 0$$

Exercise: Question 8

Actions:
„…to buy 0, 1000, 2000 or 3000 nights …“
$a_1: 0$
$a_2: 1000$
$a_3: 2000$
$a_4: 3000$

Scenarios:
„… good year, average year, bad year …“
$s_1: 3000$
$s_2: 1800$
$s_3: 900$

Also:
Cost per night: 90 EUR
Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1			
a_2			
a_3	20000	0	-90000
a_4			

$$Cost: 2000\ Nights * 90 \frac{EUR}{Night} = 180000$$

$$Revenue: 900\ Nights * 100 \frac{EUR}{Night} = 90000$$

$$Profit: 90000 - 180000 = -90000$$

Exercise: Question 8

<u>Actions:</u>

„…to buy 0, 1000, 2000 or 3000 nights …"

$a_1: 0$

$a_2: 1000$

$a_3: 2000$

$a_4: 3000$

<u>Scenarios:</u>

„… good year, average year, bad year …"

$s_1: 3000$

$s_2: 1800$

$s_3: 900$

<u>Also:</u>

Cost per night: 90 EUR

Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1	0	0	0
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

Actions:
„…to buy 0, 1000, 2000 or 3000 nights …"
$a_1: 0$
$a_2: 1000$
$a_3: 2000$
$a_4: 3000$

Scenarios:
„… good year, average year, bad year …"
$s_1: 3000$
$s_2: 1800$
$s_3: 900$

Also:
Cost per night: 90 EUR
Revenue per night: 100 EUR

	s_1	s_2	s_3
a_1	0	0	0
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Efficient alternatives?

Exercise: Question 8

<u>Actions:</u>
„…to buy 0, 1000, 2000 or 3000 nights …“
$a_1: 0$
$a_2: 1000$
$a_3: 2000$
$a_4: 3000$

<u>Scenarios:</u>
„… good year, average year, bad year …“
$s_1: 3000$
$s_2: 1800$
$s_3: 900$

<u>Also:</u>
Cost per night: 90 EUR
Revenue per night: 100 EUR

	s_1	s_2	s_3
~~a_1~~	~~0~~	~~0~~	~~0~~
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Efficient alternatives?

Exercise: Question 8

Decision based on Maximax-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

Decision based on Maximax-Rule:

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_2	10000	10000	0	10000
a_3	20000	0	-90000	20000
a_4	30000	-90000	-180000	30000

Exercise: Question 8

<u>Decision based on Maximax-Rule:</u>

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_2	10000	10000	0	10000
a_3	20000	0	-90000	20000
a_4	30000	-90000	-180000	30000

Choose alternative a_4

Exercise: Question 8

Decision based on Maximin-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

Decision based on Maximin-Rule:

	s_1	s_2	s_3	$\min_j\{e_{i,j}\}$
a_2	10000	10000	0	0
a_3	20000	0	-90000	-90000
a_4	30000	-90000	-180000	-180000

Choose alternative a_2

Exercise: Question 8

Decision based on Laplace-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

Decision based on Laplace-Rule:

	s_1	s_2	s_3	$\frac{1}{3}\sum_{j=1}^{3} e_{i,j}$
a_2	10000	10000	0	6666.66
a_3	20000	0	-90000	-23333.33
a_4	30000	-90000	-180000	-80000

Exercise: Question 8

Decision based on Laplace-Rule:

	s_1	s_2	s_3	$\frac{1}{3}\sum_{j=1}^{3} e_{i,j}$
a_2	10000	10000	0	6666.66
a_3	20000	0	-90000	-23333.33
a_4	30000	-90000	-180000	-80000

Choose alternative a_2

Exercise: Question 8

Decision based on Minimax-Regret-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000

Exercise: Question 8

Decision based on Minimax-Regret-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000
$\max_i\{e_{i,j}\}$	30000	10000	0

Exercise: Question 8

Decision based on Minimax-Regret-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000
$\max_i\{e_{i,j}\}$	30000	10000	0

Regret-Matrix:

	s_1	s_2	s_3
a_2	20000	0	0
a_3	10000	10000	90000
a_4	0	100000	180000

Exercise: Question 8

Decision based on Minimax-Regret-Rule:

	s_1	s_2	s_3
a_2	10000	10000	0
a_3	20000	0	-90000
a_4	30000	-90000	-180000
$\max_i\{e_{i,j}\}$	30000	10000	0

Regret-Matrix:

	s_1	s_2	s_3	$\max_j\{e_{i,j}\}$
a_2	20000	0	0	20000
a_3	10000	10000	90000	90000
a_4	0	100000	180000	180000

Choose alternative a_2

Decision under Risk: Definitions & Notation

<u>Various ways to distinguish between different decision situations:</u>

Level of Uncertainty

Decision under Certainty
The future is known and described by a single scenario. („deterministic“)

Decision under Uncertainty
There are multiple different scenarios for the future, however their probabilities are unknown.
Number of scenarios: $n > 1$

Decision under Risk
There are multiple different scenarios for the future, their probabilities are known.

Number of scenarios: $n > 1$
Each scenario $j = 1, \dots, n$ has a probability of $0 < p_j < 1$ with $\sum_{j=1}^{n} p_j = 1$

Decision under Risk: Definitions & Notation

Decision making under risk:

- Risk
- One criterion
- Single decision maker
- Static decision

→ There are m alternatives („actions“) $a_1, \ldots, a_m$ of which the decision maker has to take one

→ These actions can also be regarded as lotteries $l_1, \ldots, l_m$

→ There are n different scenarios $s_1, \ldots, s_n$ which may unfold and of which exactly one scenario will occur

→ The probability that a scenario s_j occurs is known and denoted as p_j

Decision under Risk: Definitions & Notation

Extension of decision matrix with probabilities p_j:

	s_1	s_2	s_3
a_1	$e_{1,1}$	$e_{1,2}$	$e_{1,3}$
a_2	$e_{2,1}$	$e_{2,2}$	$e_{2,3}$
a_3	$e_{3,1}$	$e_{3,2}$	$e_{3,3}$
a_4	$e_{4,1}$	$e_{4,2}$	$e_{4,3}$

	p_1	p_2	p_3
	s_1	s_2	s_3
a_1	$e_{1,1}$	$e_{1,2}$	$e_{1,3}$
a_2	$e_{2,1}$	$e_{2,2}$	$e_{2,3}$
a_3	$e_{3,1}$	$e_{3,2}$	$e_{3,3}$
a_4	$e_{4,1}$	$e_{4,2}$	$e_{4,3}$

It holds: $\sum_{j=1}^{n} p_j = 1$ $\quad 0 < p_j < 1$

Each row of the decision matrix under risk can be represented as a **lottery** $L_i = \left(p_1, e_{i,1}; \dots; p_n, e_{i,n}\right)$

The **Expected Value (EV)** of a lottery is: $EV(L_i) = \sum_{j=1}^{n} p_j * e_{i,j}$

Decision under Risk: Definitions & Notation

<u>Introduction of concept of (expected) utility:</u>

- Not all decision makers behave rationally

$$EV(L) = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 4 + \frac{1}{8} \cdot 8 + \cdots$$

- Different attitudes towards risk result in the selection of different lotteries
- Compare thought experiment St. Petersburg-Paradox:

  → Even though the Expected Value of the lottery is infinite, most people would exchange the lottery with a certain payout

  → The Expected Value is not always a suited method to value lotteries

  → The value of an outcome e is determined by the decision maker's personal risk attitude (captured in a utility function $u(x)$)

Nicolas Bernoulli

Daniel Bernoulli

Pictures' Source: Wikipedia

Decision under Risk: Definitions & Notation

Introduction of concept of (expected) utility:

- $u(x): x \in [e^-, e^+] \to [0,1]$

  with
  e^-: worst possible outcome
  e^+: best possible outcome

Assigns utility u in between 0 and 1 to every outcome e

- **Expected Value (EV)** of lottery L_i**:** $EV(L_i) = \sum_{j=1}^{n} p_j * e_{i,j}$

- **Expected Utility (EU)** of lottery L_i**:** $EU(L_i) = \sum_{j=1}^{n} p_j * u(e_{i,j})$

Decision under Risk: Definitions & Notation

<u>Characteristics of utility functions:</u>

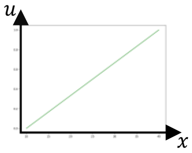
- $u(e^-) = 0$
- $u(e^+) = 1$
- $u(a) < u(b) \;\; if \; e^- \leq a < b \leq e^+$

Strictly monotonically increasing

<u>We differentiate between</u>:

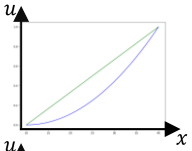
<u>Examples:</u>

- Linear utility function: $u(x) = \frac{x-10}{30}$



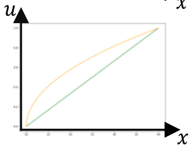
risk neutral

- Convex utility function: $u(x) = \left(\frac{x-10}{30}\right)^2$



risk seeking

- Concave utility function: $u(x) = \sqrt{\frac{x-10}{30}}$



risk averse

$x \in [10, 40], u \in [0,1]$

Decision under Risk: Definitions & Notation

Certainty Equivalent:

The **Certainty Equivalent (CE)** of a lottery L is the value for which the decision maker is indifferent between the lottery L and a certain payout of CE: $\left(1, CE(L)\right) \sim L$

Example:

Given the following utility function: $u(x) = \frac{(x-10)}{30}$

Assume you can choose between lottery L which has a payout of 40 EUR with a probability of 60% and a payout of 10 EUR with a probability of 40% and a certain payout of 28 EUR if you choose not to play the lottery. Do you choose to play the lottery or take the 28 EUR?

Step 1: Expected Utility EU of L: $EU(L) = 0.6 \cdot \frac{40-10}{30} + 0.4 \cdot \frac{10-10}{30} = 0.6$

Decision under Risk: Definitions & Notation

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Step 1: Expected Utility EU of L: $EU(L) = 0.6 * \frac{(40-10)}{30} + 0.4 * \frac{(10-10)}{30} = 0.6$

Decision under Risk: Definitions & Notation

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Example:

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Assume you can choose between lottery L which has a payout of 40 EUR with a probability of 60% and a payout of 10 EUR with a probability of 40% and a certain payout of 28 EUR if you choose not to play the lottery. Do you choose to play the lottery or take the 28 EUR?

Step 1: Expected Utility EU of L: $EU(L) = 0.6 * \frac{(40-10)}{30} + 0.4 * \frac{(10-10)}{30} = 0.6$

Step 2: Which certain payout generates the same utility? $u(x) = 0.6 \Rightarrow 3.6 = x - 10$

$28 = x$

Decision under Risk: Definitions & Notation

Certainty Equivalent:

The **Certainty Equivalent (CE)** of a lottery L is the value for which the decision maker is indifferent between the lottery L and a certain payout of CE: $\left(1, CE(L)\right) \sim L$

Example:

Given the following utility function: $u(x) = \frac{(x-10)}{30}$

Assume you can choose between lottery L which has a payout of 40 EUR with a probability of 60% and a payout of 10 EUR with a probability of 40% and a certain payout of 28 EUR if you choose not to play the lottery. Do you choose to play the lottery or take the 28 EUR?

Step 1: Expected Utility EU of L: $EU(L) = 0.6 * \frac{(40-10)}{30} + 0.4 * \frac{(10-10)}{30} = 0.6$

Step 2: Which certain payout generates the same utility? $u(x) = \frac{(x-10)}{30} = 0.6 \rightarrow \mathrm{x} = 28$

Decision under Risk: Definitions & Notation

Certainty Equivalent:

The **Certainty Equivalent (CE)** of a lottery L is the value for which the decision maker is indifferent between the lottery L and a certain payout of CE: $(1, CE(L)) \sim L$

Example:

Given the following utility function: $u(x) = \frac{(x-10)}{30}$

$\rightarrow x = 30 * u(x) + 10$

Assume you can choose between lottery L which has a payout of 40 EUR with a probability of 60% and a payout of 10 EUR with a probability of 40% and a certain payout of 28 EUR if you choose not to play the lottery. Do you choose to play the lottery or take the 28 EUR?

Step 1: Expected Utility EU of L: $EU(L) = 0.6 * \frac{(40-10)}{30} + 0.4 * \frac{(10-10)}{30} = 0.6$

Step 2: Which certain payout generates the same utility? $u(x) = \frac{(x-10)}{30} = 0.6 \rightarrow \mathrm{x} = 28$

Decision under Risk: Definitions & Notation

Certainty Equivalent:

The **Certainty Equivalent (CE)** of a lottery L is the value for which the decision maker is indifferent between the lottery L and a certain payout of CE: $\left(1, CE(L)\right) \sim L$

Example:

Given the following utility function: $u(x) = \frac{(x-10)}{30}$

Assume you can choose between lottery L which has a payout of 40 EUR with a probability of 60% and a payout of 10 EUR with a probability of 40% and a certain payout of 28 EUR if you choose not to play the lottery. Do you choose to play the lottery or take the 28 EUR?

Step 1: Expected Utility EU of L: $EU(L) = 0.6 * \frac{(40-10)}{30} + 0.4 * \frac{(10-10)}{30} = 0.6$

Step 2: Which certain payout generates the same utility? $u(x) = \frac{(x-10)}{30} = 0.6 \rightarrow \boxed{x = 28}$ Certainty Equivalent of lottery

Decision under Risk: Definitions & Notation

Certainty Equivalent:

The **Certainty Equivalent (CE)** of a lottery L is the value for which the decision maker is indifferent between the lottery L and a certain payout of CE: $\left(1, CE(L)\right) \sim L$

Example:

Given the following utility function: $u(x) = \frac{(x-10)}{30}$

Assume you can choose between lottery L which has a payout of 40 EUR with a probability of 60% and a payout of 10 EUR with a probability of 40% and a certain payout of 28 EUR if you choose not to play the lottery. Do you choose to play the lottery or take the 28 EUR?

Step 1: Expected Utility EU of L: $EU(L) = 0.6 * \frac{(40-10)}{30} + 0.4 * \frac{(10-10)}{30} = 0.6$

Step 2: Which certain payout generates the same utility? $u(x) = \frac{(x-10)}{30} = 0.6 \rightarrow \mathrm{x} = 28$

→ In this case you are indifferent between the lottery l and a certain payout of 28 EUR

Decision under Risk: Definitions & Notation

Risk Premium:

The **Risk Premium (RP)** is the difference between the expected value of the lottery and the certainty equivalent:

$$\mathrm{RP}(\mathrm{L}) = EV(L) - CE(L)$$

„… how much more (or less) the decision maker would be willing to pay to participate in the lottery compared to a risk neutral decision maker. “

Decision under Risk: Definitions & Notation

Risk Premium:

The **Risk Premium (RP)** is the difference between the expected value of the lottery and the certainty equivalent:

$$\mathrm{RP}(\mathrm{L}) = EV(L) - CE(L)$$

„… how much more (or less) the decision maker would be willing to pay to participate in the lottery compared to a risk neutral decision maker. “

- A risk neutral decision maker would be willing to pay EV(L) to participate in the lottery. (RP = 0)
- A risk averse decision maker would pay only less than the EV of the lottery. He values certainty and would take a certain payout even if CE<EV. (RP > 0)
- A risk seeking decision maker would pay more than the EV of the lottery. He values chance and sees it as a potential gain. The certain payout is CE>EV. (RP < 0)

Decision under Risk: Definitions & Notation

Risk Premium:

The **Risk Premium (RP)** is the difference between the expected value of the lottery and the certainty equivalent:

$$\mathrm{RP(L)} = EV(L) - CE(L)$$

„… how much more (or less) the decision maker would be willing to pay to participate in the lottery compared to a risk neutral decision maker. "

Example (from previous slide) with $CE(L) = 28$:

$$EV(L) = 0.6 \cdot 40 + 0.4 \cdot 10 = 28 = CE(L) \Rightarrow RP(L) = 0$$

Decision under Risk: Definitions & Notation

Risk Premium:

The **Risk Premium (RP)** is the difference between the expected value of the lottery and the certainty equivalent:

$$\mathrm{RP(L)} = EV(L) - CE(L)$$

„… how much more (or less) the decision maker would be willing to pay to participate in the lottery compared to a risk neutral decision maker. “

Example (from previous slide) with $CE(L) = 28$:

$$EV(L) = 0.6 * 40 + 0.4 * 10 = 28$$

$$RP(L) =$$

Decision under Risk: Definitions & Notation

Risk Premium:

The **Risk Premium (RP)** is the difference between the expected value of the lottery and the certainty equivalent:

$$\mathrm{RP}(\mathrm{L}) = EV(L) - CE(L)$$

„… how much more (or less) the decision maker would be willing to pay to participate in the lottery compared to a risk neutral decision maker. "

Example (from previous slide) with $CE(L) = 28$:

$$EV(L) = 0.6 * 40 + 0.4 * 10 = 28$$

$$RP(L) = 28 - 28 = 0$$

Decision under Risk: Definitions & Notation

Risk Premium:

The **Risk Premium (RP)** is the difference between the expected value of the lottery and the certainty equivalent:

$$\mathrm{RP(L)} = EV(L) - CE(L)$$

„… how much more (or less) the decision maker would be willing to pay to participate in the lottery compared to a risk neutral decision maker. “

Example (from previous slide) with $CE(L) = 28$:

$$EV(L) = 0.6 * 40 + 0.4 * 10 = 28$$

$$RP(L) = 28 - 28 = 0$$

Observation:

For the risk neutral decision maker (compare to linear utility function), the Risk Premium is 0.
His Certainty Equivalent is equal to the Expected Value of the lottery.

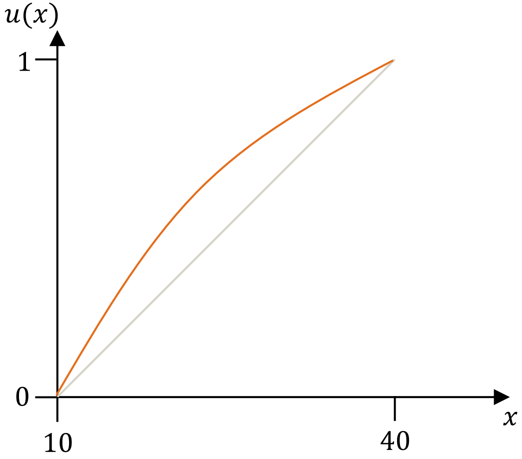
Decision under Risk: Definitions & Notation

For the same lottery, let us know assume the following utility function of a decision maker:

60%: 40 EUR
40%: 10 EUR

$$u(x) = \sqrt{\frac{x-10}{30}}$$

- What is the Certainty Equivalent of that decision maker?
- What is his Risk Premium? What is his attitude towards risk?



Decision under Risk: Definitions & Notation

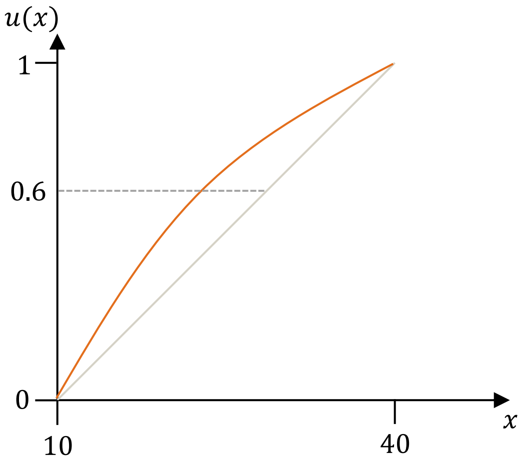
For the same lottery, let us know assume the following utility function of a decision maker:

60%: 40 EUR
40%: 10 EUR

$$u(x) = \sqrt{\frac{x-10}{30}}$$

- What is the Certainty Equivalent of that decision maker?
- What is his Risk Premium? What is his attitude towards risk?

$$EU(L) = 0.6 * \sqrt{\frac{40-10}{30}} + 0.4 * \sqrt{\frac{10-10}{30}} = 0.6$$



Decision under Risk: Definitions & Notation

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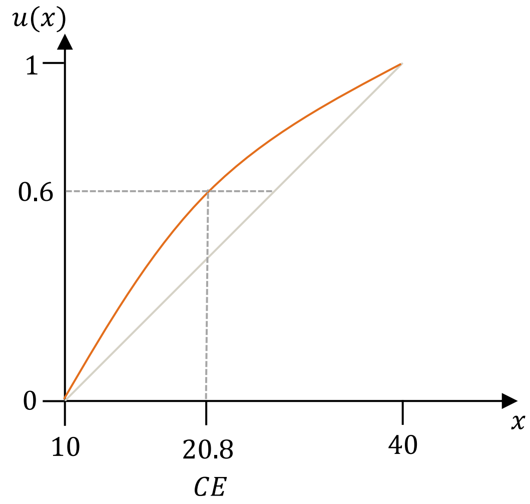
$$u(x) = \sqrt{\frac{x-10}{30}} = 0.6 \rightarrow \mathrm{x} = 20.8$$

$$CE(L) = 20.8$$



Decision under Risk: Definitions & Notation

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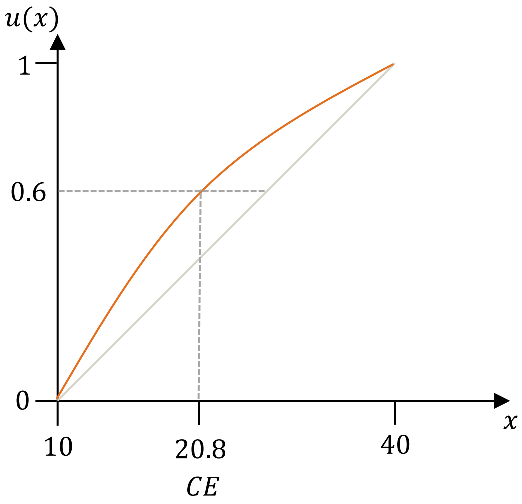

Decision under Risk: Definitions & Notation

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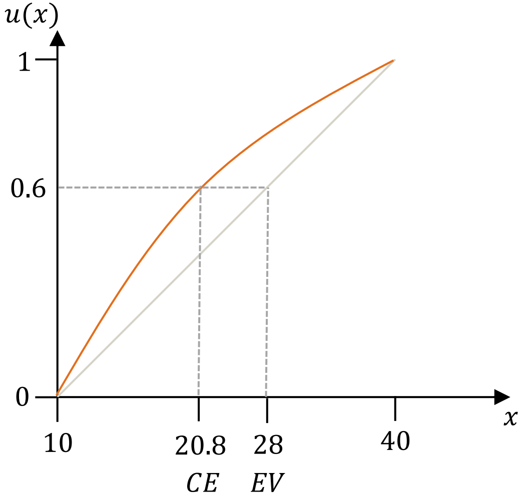
$$RP(L) = EV(L) - CE(L)$$

$$EV(L) = 0.6 * 40 + 0.4 * 10 = 28$$



Decision under Risk: Definitions & Notation

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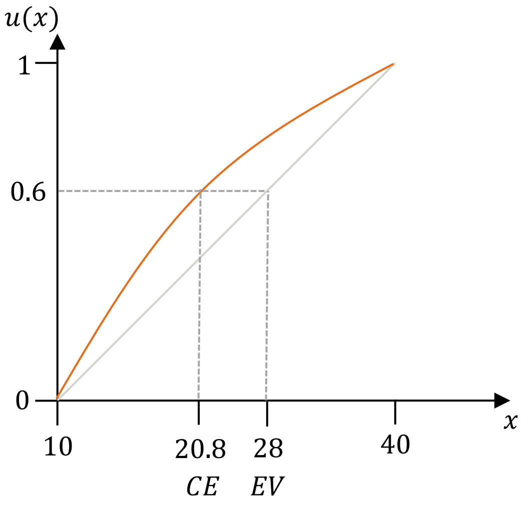
$$CE(L) = 20.8$$

$$RP(L) = EV(L) - CE(L)$$

$$EV(L) = 0.6 * 40 + 0.4 * 10 = 28$$

For a risk neutral decision maker, this is the Certainty Equivalent!



Decision under Risk: Definitions & Notation

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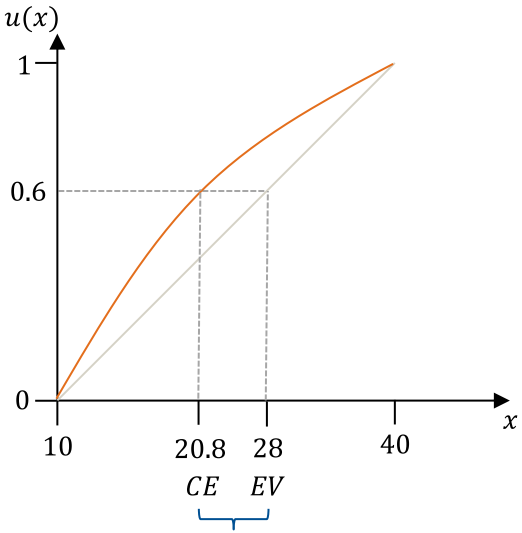
$$CE(L) = 20.8$$

$$RP(L) = EV(L) - CE(L)$$

$$EV(L) = 0.6 * 40 + 0.4 * 10 = 28$$

$$RP(L) = 28 - 20.8 = 7.2$$



$$RP = 28 - 20.8 = 7.2$$

$$\rightarrow risk\ attitude?$$

Decision under Risk: Definitions & Notation

<u>For the same lottery, let us know assume the following utility function of a decision maker:</u>

60%: 40 EUR
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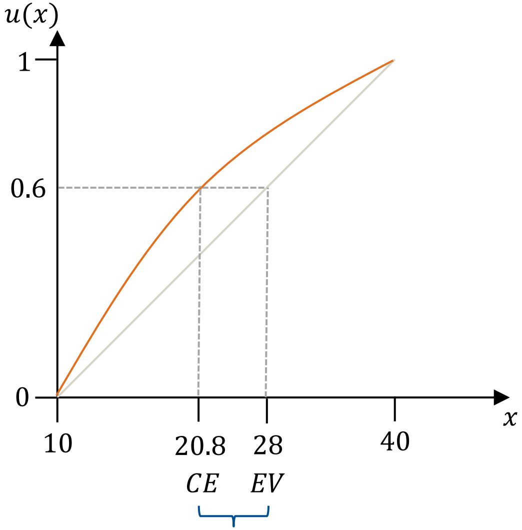


$RP = 28 - 20.8 = 7.2 > 0$

$\rightarrow risk\ averse$

Decision under Risk: Definitions & Notation

Alternative measure to determine risk attitude: **Arrow-Pratt Measure (AP)**:

Let utility function $u(x)$ be two times differentiable over the interval $[e^-, e^+]$ and the first derivative of $u(x)$ is different to zero at x for $e^- \leq x \leq e^+$. Then the **Arrow-Pratt Measure (AP)** is defined as:

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

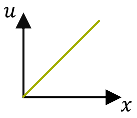
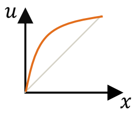
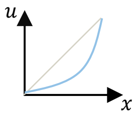
Because $u(x)$ is monotonically increasing, we know that $u'(x) > 0$.

It follows:

- $AP(x) = 0 \rightarrow risk\ neutral\ decision\ maker\ (linear\ utility\ function\ with\ u''(x) = 0)$
- $AP(x) > 0 \rightarrow risk\ averse\ decision\ maker\ (concave\ utility\ function\ with\ u''(x) < 0)$
- $AP(x) < 0 \rightarrow risk\ seeking\ decision\ maker\ (convex\ utility\ function\ with\ u''(x) > 0)$

Decision under Risk: Definitions & Notation

<u>Summary Risk Attitude:</u>

Risk neutral:	Linear utility function	$CE = EV$	$RP = 0$	$u''(x) = 0$	$AP = 0$	
Risk averse:	Concave utility function	$CE < EV$	$RP > 0$	$u''(x) < 0$	$AP > 0$	
Risk seeking:	Convex utility function	$CE > EV$	$RP < 0$	$u''(x) > 0$	$AP < 0$	

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Decision Matrix and Expected Utility

1. Derive (or be given) a utility function of a decision maker

2. Transform decision matrix of **outcomes** $e_{i,j}$ to a decision matrix of **utilities** $u(e_{i,j})$

3. Calculate for each alternative a_i the **expected utility** $EU(a_i)$:

$$EU(a_i) = \sum_{j=1}^{n} p_j * u(e_{i,j})$$

4. Select the alternative with **maximum expected utility**

→ This is nothing new, same concepts as introduced before

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 65 ff.) / Manuscript (p. 20 f.)

$$EU(a_i) = \sum_{j=1}^{n} p_j * u(e_{i,j})$$

Assume a utility function of $u(x) = \left(\frac{x-2}{8}\right)^2$

Note how $u(e^+) = 1$ and $u(e^-) = 0$ with $e^+ = 10$ and $e^- = 2$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

Outcomes $e_{i,j}$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

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	4	3	9	5	2

Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1				
	2				
	3				
	4				

Utilities $u(e_{i,j})$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

$$EU(a_i) = \sum_{j=1}^{n} p_j * u(e_{i,j})$$

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Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	0			
	2				
	3				
	4				

Utilities $u(e_{i,j})$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

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Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	0			1
	2				
	3				
	4				

Utilities $u(e_{i,j})$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

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	3	4	8	4	5
	4	3	9	5	2

Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	0		0.25	1
	2				
	3				
	4				

Utilities $u(e_{i,j})$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

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a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

Utilities $u(e_{i,j})$

	p_j	0.1	0.2	0.5	0.2
	s_j	1	2	3	4
a_i	1	0	0.00	0.25	1
	2	0.25	0.02	0.14	0.06
	3	0.06	0.56	0.06	0.14
	4	0.02	0.77	0.14	0

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

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a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2	$EU(a_i)$
	s_j	1	2	3	4	
a_i	1	0	0.00	0.25	1	
	2	0.25	0.02	0.14	0.06	
	3	0.06	0.56	0.06	0.14	
	4	0.02	0.77	0.14	0	

Utilities $u(e_{i,j})$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

$$EU(a_i) = \sum_{j=1}^{n} p_j * u(e_{i,j})$$

$$EU(a_1) = 0.1 \cdot 0 + 0.2 \cdot 0 + 0.5 \cdot 0.25 + 0.2 \cdot 1 = 0.33$$

Assume a utility function of $u(x) = \left(\frac{x-2}{8}\right)^2$

Note how $u(e^+) = 1$ and $u(e^-) = 0$ with $e^+ = 10$ and $e^- = 2$

	p_j	0.1	0.2	0.5	0.2
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a_i	1	2	2	6	10
	2	6	3	5	4
	3	4	8	4	5
	4	3	9	5	2

Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2	$EU(a_i)$
	s_j	1	2	3	4	
a_i	1	0	0.00	0.25	1	0.33
	2	0.25	0.02	0.14	0.06	
	3	0.06	0.56	0.06	0.14	
	4	0.02	0.77	0.14	0	

Utilities $u(e_{i,j})$

Decision under Risk: Definitions & Notation

Chapter 1.3.6.: Example from Lecture Slides (p. 63 ff.) / Manuscript (p. 20 f.)

$$EU(a_i) = \sum_{j=1}^{n} p_j * u(e_{i,j})$$

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	4	3	9	5	2

Outcomes $e_{i,j}$

	p_j	0.1	0.2	0.5	0.2	$EU(a_i)$
	s_j	1	2	3	4	
a_i	1	0	0.00	0.25	1	0.33
	2	0.25	0.02	0.14	0.06	0.11
	3	0.06	0.56	0.06	0.14	0.18
	4	0.02	0.77	0.14	0	0.23

Utilities $u(e_{i,j})$

Choose alternative a_1
[Highest EU]

Decision under Risk: Definitions & Notation

$\mu - \sigma -$Criterion

Making a decision based on a preference function that takes into account the expected outcome and the variance.

Expected Value of the outcome of an alternative:

$$\mu(a_i) = \sum_{j=1}^{n} p_j * e_{i,j}$$

Variance of an alternative:

$$\sigma^2(a_i) = \sum_{j=1}^{n} p_j * \left(\mu(a_i) - e_{i,j}\right)^2$$

Preference function:

$$\Phi(a_i) = \alpha * \mu(a_i) + \beta * \sigma(a_i)$$

Decision under Risk: Definitions & Notation

$\mu - \sigma -$Criterion

Making a decision based on a preference function that takes into account the expected outcome and the variance.

Expected Value of the outcome of an alternative:

$$\mu(a_i) = \sum_{j=1}^{n} p_j * e_{i,j}$$

Variance of an alternative:

$$\sigma^2(a_i) = \sum_{j=1}^{n} p_j * \left(\mu(a_i) - e_{i,j}\right)^2$$

Preference function:

$$\Phi(a_i) = \alpha * \mu(a_i) + \beta * \sigma(a_i)$$

For a risk neutral decision maker: $\beta = 0$
For a risk averse decision maker: $\beta < 0$
For a risk seeking decision maker: $\beta > 0$

Exercise: Question 16

Question 16

The utility function of Mr. Kunz is estimated as $u(x) = \sqrt{\frac{x}{10}}$. He is offered to participate in a lottery, which brings a payout of 10 Euro with a probability of 64%. Otherwise the payout is 0.

a) Determine the expected utility, the certainty equivalent and the risk premium.

b) Which attitude toward risk does Mr. Hinz have?

c) Which amount would Mr. Kunz or a risk-neutral decision maker pay at most to participate in the lottery?

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$
$$EU(L) = 0.64 * 1 + 0.36 * 0$$
$$EU(L) = 0.64$$

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$
$$EU(L) = 0.64 * 1 + 0.36 * 0$$
$$EU(L) = 0.64$$

$$CE(L) = u^{-1}(0.64)$$

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$EU(L) = 0.64 * u(10) + 0.36 * u(0)$
$EU(L) = 0.64 * 1 + 0.36 * 0$
$EU(L) = 0.64$

$CE(L) = u^{-1}(0.64)$

Inverse of utility function $u(x)$ → $u^{-1}(y)$:
„… what value of x generates a utility of 0.64?"

$u(x) = \sqrt{\frac{x}{10}}$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$EU(L) = 0.64 * u(10) + 0.36 * u(0)$
$EU(L) = 0.64 * 1 + 0.36 * 0$
$EU(L) = 0.64$

$CE(L) = u^{-1}(0.64)$

$\rightarrow u(x) = \sqrt{\frac{x}{10}}$

$\rightarrow y = \sqrt{\frac{x}{10}}$

$\rightarrow y^2 * 10 = x$

$\rightarrow y^2 * 10 = u^{-1}(y)$

Inverse of utility function $u(x)$ → $u^{-1}(y)$:
„… what value of x generates a utility of 0.64?"

$u(x) = \sqrt{\frac{x}{10}}$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$u(x) = \sqrt{\frac{x}{10}}$

64% → 10 EUR
36% → 0 EUR

$EU(L) = 0.64 * u(10) + 0.36 * u(0)$
$EU(L) = 0.64 * 1 + 0.36 * 0$
$EU(L) = 0.64$

Inverse of utility function $u(x)$ → $u^{-1}(y)$:
„… what value of x generates a utility of 0.64?"

$CE(L) = u^{-1}(0.64)$

$\rightarrow u(x) = \sqrt{\frac{x}{10}}$

$\rightarrow y = \sqrt{\frac{x}{10}}$
$\rightarrow y^2 * 10 = x$
$\rightarrow y^2 * 10 = u^{-1}(y)$

$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$
$\rightarrow u^{-1}(0.64) = 4.096$
$CE(L) = 4.096$

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$
$$EU(L) = 0.64 * 1 + 0.36 * 0$$
$$EU(L) = 0.64$$

$$CE(L) = u^{-1}(0.64)$$
$$\rightarrow u(x) = \sqrt{\frac{x}{10}}$$
$$\rightarrow y = \sqrt{\frac{x}{10}}$$
$$\rightarrow y^2 * 10 = x$$
$$\rightarrow y^2 * 10 = u^{-1}(y)$$

$$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$$
$$\rightarrow u^{-1}(0.64) = 4.096$$
$$CE(L) = 4.096$$

$$RP(L) = EV(L) - CE(L)$$

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$
$$EU(L) = 0.64 * 1 + 0.36 * 0$$
$$EU(L) = 0.64$$

$$CE(L) = u^{-1}(0.64)$$
$$\rightarrow u(x) = \sqrt{\frac{x}{10}}$$
$$\rightarrow y = \sqrt{\frac{x}{10}}$$
$$\rightarrow y^2 * 10 = x$$
$$\rightarrow y^2 * 10 = u^{-1}(y)$$

$$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$$
$$\rightarrow u^{-1}(0.64) = 4.096$$
$$CE(L) = 4.096$$

$$RP(L) = EV(L) - CE(L)$$

$$EV(L) = 0.64 * 10 + 0.36 * 0$$
$$EV(L) = 6.4$$

Exercise: Question 16

a): Expected Utility, Certainty Equivalent, Risk Premium?

$u(x) = \sqrt{\frac{x}{10}}$

64% → 10 EUR
36% → 0 EUR

$EU(L) = 0.64 * u(10) + 0.36 * u(0)$
$EU(L) = 0.64 * 1 + 0.36 * 0$
$EU(L) = 0.64$

$CE(L) = u^{-1}(0.64)$

$\rightarrow u(x) = \sqrt{\frac{x}{10}}$

$\rightarrow y = \sqrt{\frac{x}{10}}$

$\rightarrow y^2 * 10 = x$

$\rightarrow y^2 * 10 = u^{-1}(y)$

$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$

$\rightarrow u^{-1}(0.64) = 4.096$

$CE(L) = 4.096$

$RP(L) = EV(L) - CE(L)$

$EV(L) = 0.64 * 10 + 0.36 * 0$
$EV(L) = 6.4$

$RP(L) = 6.4 - 4.096$
$RP(L) = 2.304$

Exercise: Question 16

b): Risk attitude?

$EU(L) = 0.64 * u(10) + 0.36 * u(0)$
$EU(L) = 0.64 * 1 + 0.36 * 0$
$EU(L) = 0.64$

$CE(L) = u^{-1}(0.64)$

$\rightarrow u(x) = \sqrt{\frac{x}{10}}$

$\rightarrow y = \sqrt{\frac{x}{10}}$
$\rightarrow y^2 * 10 = x$
$\rightarrow y^2 * 10 = u^{-1}(y)$

$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$
$\rightarrow u^{-1}(0.64) = 4.096$
$CE(L) = 4.096$

$RP(L) = EV(L) - CE(L)$

$EV(L) = 0.64 * 10 + 0.36 * 0$
$EV(L) = 6.4$

$RP(L) = 6.4 - 4.096$
$RP(L) = 2.304$

$u(x) = \sqrt{\frac{x}{10}}$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

b): Risk attitude?

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$
$$EU(L) = 0.64 * 1 + 0.36 * 0$$
$$EU(L) = 0.64$$

$$CE(L) = u^{-1}(0.64)$$
$$\rightarrow u(x) = \sqrt{\frac{x}{10}}$$
$$\rightarrow y = \sqrt{\frac{x}{10}}$$
$$\rightarrow y^2 * 10 = x$$
$$\rightarrow y^2 * 10 = u^{-1}(y)$$

$$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$$
$$\rightarrow u^{-1}(0.64) = 4.096$$
$$CE(L) = 4.096$$

$$RP(L) = EV(L) - CE(L)$$

$$EV(L) = 0.64 * 10 + 0.36 * 0$$
$$EV(L) = 6.4$$

$$RP(L) = 6.4 - 4.096$$
$$\boxed{RP(L) = 2.304}$$

Because his Risk Premium is > 0, he is risk averse. He would pay 2.304 EUR less for the participation in the lottery compared to a risk neutral decision maker.

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

c): Which amount would he (or a risk neutral decision maker) pay at most to participate in the lottery?

$u(x) = \sqrt{\frac{x}{10}}$

64% → 10 EUR
36% → 0 EUR

$EU(L) = 0.64 * u(10) + 0.36 * u(0)$
$EU(L) = 0.64 * 1 + 0.36 * 0$
$EU(L) = 0.64$

$CE(L) = u^{-1}(0.64)$

$\rightarrow u(x) = \sqrt{\frac{x}{10}}$

$\rightarrow y = \sqrt{\frac{x}{10}}$

$\rightarrow y^2 * 10 = x$
$\rightarrow y^2 * 10 = u^{-1}(y)$

$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$
$\rightarrow u^{-1}(0.64) = 4.096$
$CE(L) = 4.096$

$RP(L) = EV(L) - CE(L)$

$EV(L) = 0.64 * 10 + 0.36 * 0$
$EV(L) = 6.4$

$RP(L) = 6.4 - 4.096$
$RP(L) = 2.304$

Exercise: Question 16

c): Which amount would he (or a risk neutral decision maker) pay at most to participate in the lottery?

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

$$EU(L) = 0.64 * u(10) + 0.36 * u(0)$$
$$EU(L) = 0.64 * 1 + 0.36 * 0$$
$$EU(L) = 0.64$$

$$CE(L) = u^{-1}(0.64)$$
$$\rightarrow u(x) = \sqrt{\frac{x}{10}}$$
$$\rightarrow y = \sqrt{\frac{x}{10}}$$
$$\rightarrow y^2 * 10 = x$$
$$\rightarrow y^2 * 10 = u^{-1}(y)$$

$$\rightarrow u^{-1}(0.64) = 0.64^2 * 10$$
$$\rightarrow u^{-1}(0.64) = 4.096$$
$$\boxed{CE(L) = 4.096}$$

$$RP(L) = EV(L) - CE(L)$$

$$EV(L) = 0.64 * 10 + 0.36 * 0$$
$$\boxed{EV(L) = 6.4}$$

$$RP(L) = 6.4 - 4.096$$
$$RP(L) = 2.304$$

Mr. Kunz would pay at most 4.096 EUR to participate in the lottery.
A risk neutral decision maker would pay at most 6.4 EUR to participate in the lottery

Exercise: Question 16

Extension: Risk Attitude via Arrow-Pratt Measure

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u(x) = \sqrt{\frac{x}{10}} = \left(\frac{x}{10}\right)^{\frac{1}{2}}$$

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

Extension: Risk Attitude via Arrow-Pratt Measure

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u(x) = \sqrt{\frac{x}{10}} = \left(\frac{x}{10}\right)^{\frac{1}{2}}$$

$$u'(x) = \frac{1}{2} * \left(\frac{x}{10}\right)^{-\frac{1}{2}} * \frac{1}{10}$$

$$\left[\dots = \frac{1}{2} * \frac{1}{10} * \frac{\sqrt{10}}{\sqrt{x}} = \frac{1}{2} * \frac{1}{\sqrt{10} * \sqrt{x}}\right]$$

Exercise: Question 16

Extension: Risk Attitude via Arrow-Pratt Measure

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u(x) = \sqrt{\frac{x}{10}} = \left(\frac{x}{10}\right)^{\frac{1}{2}}$$

$$u'(x) = \frac{1}{2} * \left(\frac{x}{10}\right)^{-\frac{1}{2}} * \frac{1}{10}$$

$$\left[\ldots = \frac{1}{2} * \frac{1}{10} * \frac{\sqrt{10}}{\sqrt{x}} = \frac{1}{2} * \frac{1}{\sqrt{10} * \sqrt{x}}\right]$$

$$u''(x) = -\frac{1}{4} * \left(\frac{x}{10}\right)^{-\frac{3}{2}} * \frac{1}{10} * \frac{1}{10}$$

$$\left[\ldots = -\frac{1}{4} * \frac{1}{10} * \frac{1}{10} * \frac{\sqrt{10}^3}{\sqrt{x}^3} = -\frac{1}{4} * \frac{1}{\sqrt{10} * \sqrt{x}^3}\right]$$

Exercise: Question 16

Extension: Risk Attitude via Arrow-Pratt Measure

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u(x) = \sqrt{\frac{x}{10}} = \left(\frac{x}{10}\right)^{\frac{1}{2}}$$

$$u'(x) = \frac{1}{2} * \left(\frac{x}{10}\right)^{-\frac{1}{2}} * \frac{1}{10}$$

$$\left[\ldots = \frac{1}{2} * \frac{1}{10} * \frac{\sqrt{10}}{\sqrt{x}} = \frac{1}{2} * \frac{1}{\sqrt{10} * \sqrt{x}}\right]$$

$$u''(x) = -\frac{1}{4} * \left(\frac{x}{10}\right)^{-\frac{3}{2}} * \frac{1}{10} * \frac{1}{10}$$

$$\left[\ldots = -\frac{1}{4} * \frac{1}{10} * \frac{1}{10} * \frac{\sqrt{10}^3}{\sqrt{x}^3} = -\frac{1}{4} * \frac{1}{\sqrt{10} * \sqrt{x}^3}\right]$$

$$AP(x) = -\frac{u''(x)}{u'(x)} = -\frac{\left(-\frac{1}{4} * \left(\frac{x}{10}\right)^{-\frac{3}{2}} * \frac{1}{10} * \frac{1}{10}\right)}{\left(\frac{1}{2} * \left(\frac{x}{10}\right)^{-\frac{1}{2}} * \frac{1}{10}\right)} = \frac{1}{2 * x}$$

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

Exercise: Question 16

Extension: Risk Attitude via Arrow-Pratt Measure

$$u(x) = \sqrt{\frac{x}{10}}$$

64% → 10 EUR
36% → 0 EUR

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u(x) = \sqrt{\frac{x}{10}} = \left(\frac{x}{10}\right)^{\frac{1}{2}}$$

$$u'(x) = \frac{1}{2} * \left(\frac{x}{10}\right)^{-\frac{1}{2}} * \frac{1}{10} \qquad \left[\ldots = \frac{1}{2} * \frac{1}{10} * \frac{\sqrt{10}}{\sqrt{x}} = \frac{1}{2} * \frac{1}{\sqrt{10} * \sqrt{x}}\right]$$

$$u''(x) = -\frac{1}{4} * \left(\frac{x}{10}\right)^{-\frac{3}{2}} * \frac{1}{10} * \frac{1}{10} \qquad \left[\ldots = -\frac{1}{4} * \frac{1}{10} * \frac{1}{10} * \frac{\sqrt{10}^3}{\sqrt{x}^3} = -\frac{1}{4} * \frac{1}{\sqrt{10} * \sqrt{x}^3}\right]$$

$$AP(x) = -\frac{u''(x)}{u'(x)} = -\frac{\left(-\frac{1}{4} * \left(\frac{x}{10}\right)^{-\frac{3}{2}} * \frac{1}{10} * \frac{1}{10}\right)}{\left(\frac{1}{2} * \left(\frac{x}{10}\right)^{-\frac{1}{2}} * \frac{1}{10}\right)} = \boxed{\frac{1}{2 * x}}$$

AP(x) > 0 for all x > 0.
Therefore risk averse.

Exercise: Question 21

Question 21

A decision maker's preference is described by the utility function $u(x) = x^a$ (with $x \geq 0$ und $a > 0$).

a) Use the Arrow-Pratt measure to specify for which values of a the decision maker is

(1) Risk-neutral

(2) Risk-averse

(3) Risk-seeking

b) The decision maker has the opportunity to participate in a lottery which brings a payout of either 1 Euro or 0 Euro, each with a probability of 50%.

(1) How high is the certainty equivalent of the decision maker depending on a?

(2) Assume that $a = \frac{3}{4}$. Will the decision maker pay 30 cents for his/her participation on the lottery?

Exercise: Question 21

a): Use the Arrow-Pratt Measure to determine for which values of a the decision maker is:

- risk neutral
- risk averse
- risk seeking

$$u(x) = x^a \qquad \text{with x} \geq 0 \text{ and a} > 0$$

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

Exercise: Question 21

a): Use the Arrow-Pratt Measure to determine for which values of a the decision maker is:

- risk neutral
- risk averse
- risk seeking

$$u(x) = x^a \quad \text{with } \mathrm{x} \geq 0 \text{ and } \mathrm{a} > 0$$

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u'(x) = \mathrm{a} * \mathrm{x}^{\mathrm{a}-1}$$

Exercise: Question 21

a): Use the Arrow-Pratt Measure to determine for which values of a the decision maker is:

- risk neutral
- risk averse
- risk seeking

$$u(x) = x^a \qquad \text{with } \mathrm{x} \geq 0 \text{ and } \mathrm{a} > 0$$

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u'(x) = \mathrm{a} * \mathrm{x}^{\mathrm{a}-1}$$

$$u''(x) = (\mathrm{a} - 1) * \mathrm{a} * \mathrm{x}^{\mathrm{a}-2}$$

Exercise: Question 21

a): Use the Arrow-Pratt Measure to determine for which values of a the decision maker is:

- risk neutral
- risk averse
- risk seeking

$u(x) = x^a \quad \text{with x} \geq 0 \text{ and a} > 0$

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u'(x) = \mathrm{a} * \mathrm{x}^{\mathrm{a}-1}$$

$$u''(x) = (\mathrm{a}-1) * \mathrm{a} * \mathrm{x}^{\mathrm{a}-2}$$

$$AP(x) = -\frac{u''(x)}{u'(x)} = -\frac{\left((\mathrm{a}-1) * \mathrm{a} * \mathrm{x}^{\mathrm{a}-2}\right)}{(\mathrm{a} * \mathrm{x}^{\mathrm{a}-1})} = -\frac{(a-1) * a * x^a * x^{-2}}{a * x^a * x^{-1}} = \frac{1-a}{x}$$

Exercise: Question 21

a): Use the Arrow-Pratt Measure to determine for which values of a the decision maker is:

- risk neutral
- risk averse
- risk seeking

$$u(x) = x^a \quad \text{with } x \geq 0 \text{ and } a > 0$$

$$AP(x) = -\frac{u''(x)}{u'(x)}$$

$$u'(x) = a * x^{a-1}$$

$$u''(x) = (a-1) * a * x^{a-2}$$

$$AP(x) = -\frac{u''(x)}{u'(x)} = -\frac{\left((a-1) * a * x^{a-2}\right)}{(a * x^{a-1})} = -\frac{(a-1) * a * x^a * x^{-2}}{a * x^a * x^{-1}} = \frac{1-a}{x}$$

Risk neutral when $AP(x) = 0$
→ $a = 1$

Risk averse when $AP(x) > 0$
→ $a < 1$

Risk seeking when $AP(x) < 0$
→ $a > 1$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:
Payout of 1 EUR with a probability of 50%
Payout of 0 EUR with a probability of 50%

(1) How high is the Certainty Equivalent of the decision maker depending on a?

$u(x) = x^a \quad \text{with } x \geq 0 \text{ and } a > 0$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:
Payout of 1 EUR with a probability of 50%
Payout of 0 EUR with a probability of 50%

(1) How high is the Certainty Equivalent of the decision maker depending on a?

$u(x) = x^a \quad \text{with } x \geq 0 \text{ and } a > 0$

$EU(L) = 0.5 * u(1) + 0.5 * u(0)$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:
Payout of 1 EUR with a probability of 50%
Payout of 0 EUR with a probability of 50%

(1) How high is the Certainty Equivalent of the decision maker depending on a?

$u(x) = x^a \quad \text{with x} \geq 0 \text{ and a} > 0$

$$EU(L) = 0.5 * u(1) + 0.5 * u(0)$$
$$EU(L) = 0.5 * 1^a + 0.5 * 0^a$$
$$EU(L) = 0.5 * 1 + 0.5 * 0$$
$$EU(L) = 0.5$$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:
Payout of 1 EUR with a probability of 50%
Payout of 0 EUR with a probability of 50%

(1) How high is the Certainty Equivalent of the decision maker depending on a?

$u(x) = x^a \quad \text{with x} \geq 0 \text{ and a} > 0$

$$EU(L) = 0.5 * u(1) + 0.5 * u(0)$$
$$EU(L) = 0.5 * 1^a + 0.5 * 0^a$$
$$EU(L) = 0.5 * 1 + 0.5 * 0$$
$$EU(L) = 0.5$$

$$CE(L) = u^{-1}(0.5)$$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:
Payout of 1 EUR with a probability of 50%
Payout of 0 EUR with a probability of 50%

(1) How high is the Certainty Equivalent of the decision maker depending on a?

$u(x) = x^a \quad \text{with } x \geq 0 \text{ and } a > 0$

$$EU(L) = 0.5 * u(1) + 0.5 * u(0)$$
$$EU(L) = 0.5 * 1^a + 0.5 * 0^a$$
$$EU(L) = 0.5 * 1 + 0.5 * 0$$
$$EU(L) = 0.5$$

$$CE(L) = u^{-1}(0.5)$$
$$\rightarrow u(x) = x^a$$
$$\rightarrow u^{-1}(y) = y^{\frac{1}{a}}$$
$$CE(L) = 0.5^{\frac{1}{a}}$$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:

Payout of 1 EUR with a probability of 50%

Payout of 0 EUR with a probability of 50%

(2) Assume now that $a = \frac{3}{4}$. Will the decision maker pay 30 cents for the participation in the lottery?

$u(x) = x^a \quad \text{with x} \geq 0 \text{ and a} > 0$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:

Payout of 1 EUR with a probability of 50%

Payout of 0 EUR with a probability of 50%

(2) Assume now that $a = \frac{3}{4}$. Will the decision maker pay 30 cents for the participation in the lottery?

$u(x) = x^a \qquad \text{with } x \geq 0 \text{ and } a > 0$

(from previous task)

$CE(L) = 0.5^{\frac{1}{a}}$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:

Payout of 1 EUR with a probability of 50%

Payout of 0 EUR with a probability of 50%

(2) Assume now that $a = \frac{3}{4}$. Will the decision maker pay 30 cents for the participation in the lottery?

$u(x) = x^a \quad \text{with x} \geq 0 \text{ and a} > 0$

(from previous task)

$CE(L) = 0.5^{\frac{1}{a}}$

$CE(L) = 0.5^{\frac{4}{3}}$

$CE(L) = 0.3968$

Exercise: Question 21

b) The decision maker has the opportunity to participate in the following lottery:
Payout of 1 EUR with a probability of 50%
Payout of 0 EUR with a probability of 50%

(2) Assume now that $a = \frac{3}{4}$. Will the decision maker pay 30 cents for the participation in the lottery?

$u(x) = x^a \quad \text{with x} \geq 0 \text{ and a} > 0$

(from previous task)

$CE(L) = 0.5^{\frac{1}{a}}$

$CE(L) = 0.5^{\frac{4}{3}}$

$CE(L) = 0.3968$

He would participate in the lottery. His willingness to pay for the lottery (0.3968) is higher than the required payment of 0.30.

Exercise: Winter Term 2014 / 2015 Q 4.2

4.2 (7P) Lottery L provides an outcome of $138,25$ with probability 75% and an outcome of b, $100 \leq b < 152$, with probability 25%. $u(x) = \frac{\sqrt{(x+44)}}{z} - y$ defined for outcomes $100 \leq x \leq 152$ is the utility function of the decision maker. The certain equivalent of the decision maker for L is $\frac{8633}{64}$. By rounding up all number to 2 internal decimal places, calculate the following:

a) Determine the values of y and z of the utility function. Motivate your answer. (3P)

b) Determine the value b of the decision maker by using utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer. (4P)

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25

25% → b, with $100 \leq b \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$"

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25

25% → b, with $100 \leq b \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$"

$u(100) = 0$
$u(152) = 1$

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq \mathrm{b} \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$“

$$u(100) = 0$$
$$u(152) = 1$$

$$u(100) = 0$$

$$0 = \frac{\sqrt{100 + 44}}{z} - y$$

$$u(x) = \frac{\sqrt{x + 44}}{z} - y$$

75% → 138.25

25% → b, with $100 \leq b \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \le b \le 152$

„…$u(x)$ *defined for outcomes* $100 \le x \le 152$“

$$u(100) = 0$$
$$u(152) = 1$$

$$u(100) = 0$$

$$0 = \frac{\sqrt{100+44}}{z} - y$$

$$u(152) = 1$$

$$1 = \frac{\sqrt{152+44}}{z} - y$$

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$"

$u(100) = 0$
$u(152) = 1$

$u(100) = 0$

$$0 = \frac{\sqrt{100+44}}{z} - y$$

$u(152) = 1$

$$1 = \frac{\sqrt{152+44}}{z} - y$$

Solve for z and y

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$"

$u(100) = 0$
$u(152) = 1$

$u(100) = 0$

$$0 = \frac{\sqrt{100+44}}{z} - y$$

$u(152) = 1$

$$1 = \frac{\sqrt{152+44}}{z} - y$$

$$y = \frac{\sqrt{100+44}}{z}$$

Solve for z and y

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$“

$u(100) = 0$
$u(152) = 1$

$u(100) = 0$

$$0 = \frac{\sqrt{100+44}}{z} - y$$

$$y = \frac{\sqrt{100+44}}{z}$$

$u(152) = 1$

$$1 = \frac{\sqrt{152+44}}{z} - y$$

$$1 = \frac{\sqrt{152+44}}{z} - \frac{\sqrt{100+44}}{z}$$

Solve for z and y

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$“

$u(100) = 0$
$u(152) = 1$

$u(100) = 0$

$$0 = \frac{\sqrt{100+44}}{z} - y$$

$$y = \frac{\sqrt{100+44}}{z}$$

$u(152) = 1$

$$1 = \frac{\sqrt{152+44}}{z} - y$$

$$1 = \frac{\sqrt{152+44}}{z} - \frac{\sqrt{100+44}}{z}$$

$$z = \sqrt{196} - \sqrt{144} = 2$$

Solve for z and y

Exercise: Winter Term 2014 / 2015 Q 4.2

a) Determine the values of z and y of the utility function. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

„…$u(x)$ *defined for outcomes* $100 \leq x \leq 152$“

$$u(100) = 0$$
$$u(152) = 1$$

$$u(100) = 0$$
$$0 = \frac{\sqrt{100+44}}{z} - y$$

$$u(152) = 1$$
$$1 = \frac{\sqrt{152+44}}{z} - y$$

$$y = \frac{\sqrt{100+44}}{z}$$

$$1 = \frac{\sqrt{152+44}}{z} - \frac{\sqrt{100+44}}{z} \quad \blacktriangleright \quad z = \sqrt{196} - \sqrt{144} = 2 \quad \blacktriangleright \quad y = \frac{\sqrt{100+44}}{2} = 6$$

Solve for z and y

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x + 44}}{z} - y$$

75% → 138.25

25% → b, with $100 \leq \text{b} \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x + 44}}{2} - 6$$

$$EU(L) = 0.75 * u(138.25) + 0.25 * u(b)$$

$$u(x) = \frac{\sqrt{x + 44}}{z} - y$$

75% → 138.25

25% → b, with $100 \leq b \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

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$$EU(L) = 0.75 * u(138.25) + 0.25 * u(b)$$

We know that for the Certainty Equivalent, the decision maker is indifferent between the lottery and the Certainty Equivalent.
We use this knowledge to derive the Expected Utility.
„What is the expected utility of the certainty equivalent?“

Exercise: Winter Term 2014 / 2015 Q 4.2

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We know that for the Certainty Equivalent, the decision maker is indifferent between the lottery and the Certainty Equivalent.
We use this knowledge to derive the Expected Utility.
„What is the expected utility of the certainty equivalent?"

$$CE(L) = u^{-1}(EU(L))$$
$$\rightarrow u(CE(L)) = EU(L)$$

$$EU(L) = \frac{\sqrt{\frac{8633}{64} + 44}}{2} - 6 = 0.6875$$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

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$$CE(L) = u^{-1}(EU(L))$$
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$$EU(L) = 0.75 * u(138.25) + 0.25 * u(b)$$
$$0.6875 = 0.75 * u(138.25) + 0.25 * u(b)$$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

$$u(x) = \frac{\sqrt{x+44}}{2} - 6$$

$$EU(L) = 0.75 * u(138.25) + 0.25 * u(b)$$

We know that for the Certainty Equivalent, the decision maker is indifferent between the lottery and the Certainty Equivalent.
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$$CE(L) = u^{-1}(EU(L))$$
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$$EU(L) = \frac{\sqrt{\frac{8633}{64} + 44}}{2} - 6 = 0.6875$$

$$EU(L) = 0.75 * u(138.25) + 0.25 * u(b)$$
$$0.6875 = 0.75 * u(138.25) + 0.25 * u(b)$$
$$0.6875 = 0.75 * \left(\frac{\sqrt{138.25+44}}{2} - 6\right) + 0.25 * \left(\frac{\sqrt{b+44}}{2} - 6\right)$$
$$b = 125$$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

Risk Premium?

$$u(x) = \frac{\sqrt{x+44}}{z} - y$$

75% → 138.25

25% → $b, \text{with } 100 \leq \text{b} \leq 152$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x + 44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

Risk Premium?

$$RP(L) = EV(L) - CE(L)$$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x + 44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

Risk Premium?

$$RP(L) = EV(L) - CE(L)$$

$$RP(L) = (0.75 * 138.25 + 0.25 * 125) - \frac{8633}{64}$$

$$RP(L) = \frac{3}{64}$$

Exercise: Winter Term 2014 / 2015 Q 4.2

b) Determine the value of b of the decision maker by using the utility function $u(x)$ with parameters $y = 6$ and $z = 2$. Determine the risk premium. Motivate your answer.

$$u(x) = \frac{\sqrt{x + 44}}{z} - y$$

75% → 138.25
25% → b, with $100 \leq b \leq 152$

Risk Premium?

$$RP(L) = EV(L) - CE(L)$$

$$RP(L) = (0.75 * 138.25 + 0.25 * 125) - \frac{8633}{64}$$

$$RP(L) = \frac{3}{64}$$

[risk averse decision maker because RP > 0]

Exercise: Question 27

Question 27

Mr. Bert wants to invest his savings of 100 Euro in securities for one year. There are three different alternatives available, each at the current market price of 100 Euro. The expected prices after one year are listed in the table below:

	$p_1 = 0.2$	$p_2 = 0.5$	$p_3 = 0.2$	$p_4 = 0.1$
	s_1	s_2	s_3	s_4
Security A	80	120	130	90
Security B	110	120	100	120
Security C	90	120	115	80

a) Determine the expected outcome and the variance of each alternative.

b) Depict each alternative in a μ–σ–diagram.

c) Assume that Mr. Bert behaves after the preference function $\Phi(a_i) = \frac{3}{2} \cdot \mu(a_i) + 2 \cdot \sigma(a_i)$.

(1) What is Mr. Bert's attitude toward risk?

(2) Which alternative would Mr. Bert prefer?

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4
Security A	80	120	130	90
Security B	110	120	100	120
Security C	90	120	115	80

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90		
Security B	110	120	100	120		
Security C	90	120	115	80		

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90		
Security B	110	120	100	120		
Security C	90	120	115	80		

expected outcome:

$$\mu(a_i) = \sum_{j=1}^{n} p_j * e_{i,j}$$

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90	111	
Security B	110	120	100	120		
Security C	90	120	115	80		

expected outcome:

$$\mu(a_i) = \sum_{j=1}^{n} p_j * e_{i,j}$$

For Security A: $\mu(a_1) = 0.2 * 80 + 0.5 * 120 + 0.2 * 130 + 0.1 * 90 = 111$

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90	111	
Security B	110	120	100	120	114	
Security C	90	120	115	80	109	

expected outcome:

$$\mu(a_i) = \sum_{j=1}^{n} p_j * e_{i,j}$$

For Security A: $\mu(a_1) = 0.2 * 80 + 0.5 * 120 + 0.2 * 130 + 0.1 * 90 = 111$
For Security B: $\mu(a_2) = 0.2 * 110 + 0.5 * 120 + 0.2 * 100 + 0.1 * 120 = 114$
For Security C: $\mu(a_3) = 0.2 * 90 + 0.5 * 120 + 0.2 * 115 + 0.1 * 90 = 109$

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90	111	
Security B	110	120	100	120	114	
Security C	90	120	115	80	109	

variance:

$$\sigma^2(a_i) = \sum_{j=1}^{n} p_j * \left(\mu(a_i) - e_{i,j}\right)^2 \qquad or\ also:\ \sigma^2(a_i) = \left(\sum_{j=1}^{n} p_j * e_{i,j}^2\right) - \mu(a_i)^2$$

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90	111	349
Security B	110	120	100	120	114	
Security C	90	120	115	80	109	

variance:

$$\sigma^2(a_i) = \sum_{j=1}^{n} p_j * \left(\mu(a_i) - e_{i,j}\right)^2 \qquad or\ also\colon\ \sigma^2(a_i) = \left(\sum_{j=1}^{n} p_j * e_{i,j}^2\right) - \mu(a_i)^2$$

For Security A: $\sigma^2(a_1) = 0.2 * 80^2 + 0.5 * 120^2 + 0.2 * 130^2 + 0.1 * 90^2 - 111^2 = 349$

Exercise: Question 27

a) Determine the expected outcome and variance for each alternative.

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90	111	349
Security B	110	120	100	120	114	64
Security C	90	120	115	80	109	224

variance:

$$\sigma^2(a_i) = \sum_{j=1}^{n} p_j * \left(\mu(a_i) - e_{i,j}\right)^2 \qquad or\ also\text{:}\ \sigma^2(a_i) = \left(\sum_{j=1}^{n} p_j * e_{i,j}^2\right) - \mu(a_i)^2$$

For Security A: $\sigma^2(a_1) = 0.2 * 80^2 + 0.5 * 120^2 + 0.2 * 130^2 + 0.1 * 90^2 - 111^2 = 349$

For Security B: $\sigma^2(a_2) = 0.2 * 110^2 + 0.5 * 120^2 + 0.2 * 100^2 + 0.1 * 120^2 - 114^2 = 64$

For Security C: $\sigma^2(a_3) = 0.2 * 90^2 + 0.5 * 120^2 + 0.2 * 115^2 + 0.1 * 90^2 - 109^2 = 224$

Exercise: Question 27

b) Depict each alternative in a $\mu - \sigma$ –Diagram

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$
Security A	80	120	130	90	111	349
Security B	110	120	100	120	114	64
Security C	90	120	115	80	109	224

Exercise: Question 27

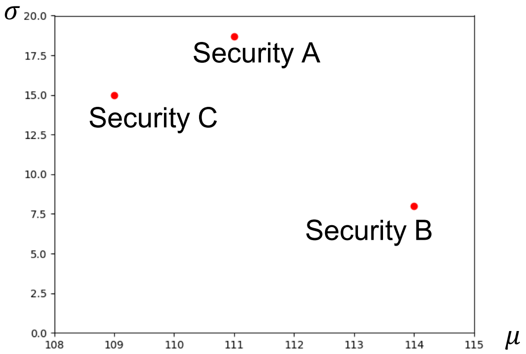
b) Depict each alternative in a $\mu - \sigma$ –Diagram

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

Exercise: Question 27

b) Depict each alternative in a $\mu - \sigma$ –Diagram

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97



Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

1) What is his attitude towards risk?

Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$	$p = 0.5$	$p = 0.2$	$p = 0.1$			
	s_1	s_2	s_3	s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

1) What is his attitude towards risk?

- Because of the decision maker‘s preference function of $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$, he values a higher variance. He is risk seeking.
- More generally: $\phi(a_i) = \alpha * \mu(a_i) + \beta * \sigma(a_i)$.
- When $\beta > 0$, the decision maker depicts risk seeking behavior.
- Analogously: When $\beta = 0$ → risk neutral behavior
- Analogously: When $\beta < 0$ → risk averse behavior

Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

2) Which alternative does he prefer?

Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

2) Which alternative does he prefer?
   - Evaluate preference function $\phi(a_i)$ for all alternatives. Choose the one alternative that yields the highest preference value:

Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

2) Which alternative does he prefer?

- Evaluate preference function $\phi(a_i)$ for all alternatives. Choose the one alternative that yields the highest preference value:
- Security A: $\phi(a_1) = \frac{3}{2} * \mu(a_1) + 2 * \sigma(a_1) = \frac{3}{2} * 111 + 2 * 18.68 = 203.86$

Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$	$p = 0.5$	$p = 0.2$	$p = 0.1$			
	s_1	s_2	s_3	s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

2) Which alternative does he prefer?

- Evaluate preference function $\phi(a_i)$ for all alternatives. Choose the one alternative that yields the highest preference value:
- Security A: $\phi(a_1) = \frac{3}{2} * \mu(a_1) + 2 * \sigma(a_1) = \frac{3}{2} * 111 + 2 * 18.68 = 203.86$
- Security B: $\phi(a_2) = \frac{3}{2} * \mu(a_2) + 2 * \sigma(a_2) = \frac{3}{2} * 114 + 2 * 8 = 187$
- Security C: $\phi(a_3) = \frac{3}{2} * \mu(a_3) + 2 * \sigma(a_3) = \frac{3}{2} * 109 + 2 * 147.97 = 193.44$

Exercise: Question 27

c) Assume that he bases his decision on the preference function $\phi(a_i) = \frac{3}{2} * \mu(a_i) + 2 * \sigma(a_i)$

	$p = 0.2$ s_1	$p = 0.5$ s_2	$p = 0.2$ s_3	$p = 0.1$ s_4	$\mu(a_i)$	$\sigma^2(a_i)$	$\sigma(a_i)$
Security A	80	120	130	90	111	349	18.68
Security B	110	120	100	120	114	64	8
Security C	90	120	115	80	109	224	14.97

2) Which alternative does he prefer?
- Evaluate preference function $\phi(a_i)$ for all alternatives. Choose the one alternative that yields the highest preference value:
- Security A: $\phi(a_1) = \frac{3}{2} * \mu(a_1) + 2 * \sigma(a_1) = \frac{3}{2} * 111 + 2 * 18.68 = 203.86$
- Security B: $\phi(a_2) = \frac{3}{2} * \mu(a_2) + 2 * \sigma(a_2) = \frac{3}{2} * 114 + 2 * 8 = 187$
- Security C: $\phi(a_3) = \frac{3}{2} * \mu(a_3) + 2 * \sigma(a_3) = \frac{3}{2} * 109 + 2 * 147.97 = 193.44$

He chooses alternative a_1.

Thank you!

Baturhan Bayraktar