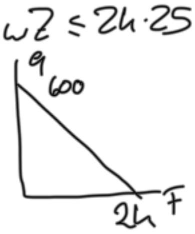
Dr. Christian Feilcke
TUM School of Management

Exercise 2: Consumption and Demand

Problem 1 (*Budget Constraint*)

Consider an individual who allocates $Z = 24$ hours on labor $L \geq 0$ and free time $F \geq 0$. Per hour of labor, the individual earns the wage rate $w = 25$. She spends her entire earned income wL on a consumption good, the quantity of which is denoted by q and the price of which is given by $p = 1$.

(a) Specify the individual's budget constraint in terms of potential income wZ, and draw her budget line in a diagram with the quantity of free time F on the horizontal axis and the quantity of the consumption good q on the vertical axis. $wZ \leq 24 \cdot 25 = 600$



(b) How does the individual's budget line change if

(i) an income tax reduces the wage rate to $(1-t)w = 20$? slope $+\frac{t}{w}$; closer to 0, max 480

(ii) a consumption tax raises the price of the consumption good to $(1+\tau)p = 1.25$? again, flattens, max is 480

(iii) earned income below a threshold $wL < 200$ is subsidized with a social transfer $S = 200 - wL$? slope = 0 starting at $F>16$, $q=200$

Problem 2 (*Assumptions on Preferences*)

Consider an individual who derives utility from the consumption of two goods, apples and oranges. Assume that she is indifferent between consumption bundle A (8 apples, 2 oranges) and consumption bundle B (2 apples, 8 oranges) and that she prefers consumption bundle B to consumption bundle C (6 apples, 6 oranges). Determine, whether the four assumptions on preferences (completeness,1 transitivity,2 monotonicity,3 and convexity4) can hold together in this case.

$A=(8,2)$
$B=(2,8)$
$C=(6,6)$
$A\sim B$, $B\succ C$

① → ✓ All bundles compared (that we know of)
② → ✓ $A=B>C \Rightarrow A>C$
③ → × $A_o < C_o$, $A>C$; $B_A<C_A$, $B>C$
④ → × C is Mix of A,B, but less preferred

– $A\succ C$ or $A\prec C$ or $A\sim C$
– $A\sim B$ and $B\prec C \Rightarrow A\succ C$
–
–

Problem 3 (*Individual Demand*)

Consider a utility-maximizing individual with a given income $y > 0$. The individual's utility function is given by

$$U(q_1, q_2) = q_1^{\frac{1}{2}} + q_2^{\frac{1}{2}},$$

where $q_1 \geq 0$ and $q_2 \geq 0$ denote the quantities of good 1 and good 2, respectively. The goods prices are given by $p_1 > 0$ and $p_2 > 0$, respectively.

(a) Determine the individual demand for each good as a function of prices and income.

3

$$\mathcal{L} = U(q_1, q_2) + \lambda(y - q_1 p_1 - q_2 p_2) \quad = \cdot$$

$$\frac{\partial \mathcal{L}}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = \frac{1}{2} q_1^{-\frac{1}{2}} - \lambda p_1 = 0 \Rightarrow \lambda = \frac{1}{2p_1} q_1^{-\frac{1}{2}}$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = \frac{\partial U}{\partial q_2} - \lambda p_2 = \frac{1}{2} q_2^{-\frac{1}{2}} - \lambda p_2 = 0 \Rightarrow \lambda = \frac{1}{2p_2} q_2^{-\frac{1}{2}}$$

$$\lambda = \lambda$$

$$\frac{1}{2p_1} q_1^{-\frac{1}{2}} = \frac{1}{2p_2} q_2^{-\frac{1}{2}}$$

$$\Rightarrow p_1 \cdot q_1^{-\frac{1}{2}} = p_2 \cdot q_2^{-\frac{1}{2}}$$

$$\Rightarrow \left(\frac{q_1}{q_2}\right)^{-\frac{1}{2}} = \frac{p_2}{p_1} \Rightarrow q_2 = q_1 \left(\frac{p_1}{p_2}\right)^2$$

Now sub. in budget constraint

$$q_2 = q_1\left(\frac{p_1}{p_2}\right)^2$$

$$y = p_1 q_1 + p_2 q_2$$

$$y = p_1 q_1 + p_2\left(q_1\left(\frac{p_1}{p_2}\right)^2\right) = p_1 q_1 + q_1 \frac{p_1^2}{p_2}$$

$$\Rightarrow q_1 = \frac{y}{p_1\left(1 + \frac{p_1}{p_2}\right)}$$

$$q_2 = \frac{y}{p_1\left(1+\frac{p_1}{p_2}\right)}\left(\frac{p_1}{p_2}\right)^2$$

...GPT says sub q_1 back into budget constraint and get

$$q_2 = \frac{y - p_1 q_1}{p_2}$$

$\Rightarrow$ generalized

$$q_i(p_i, p_j, y) = \frac{y \cdot p_j}{p_i p_j + p_i^2}$$

b) - normal / inferior good

$$\frac{\partial q_i}{\partial y} = \frac{p_j}{p_i p_j + p_i^2} > 0 \rightarrow \text{both goods normal}$$

giffen goods must be inferior anyway

- ordinary / gifty good

$$\frac{\partial q_i}{\partial p_i} = -\frac{y p_j (p_j + 2p_i)}{(p_i p_j + p_i^2)^2} < 0 \rightarrow \text{both goods ordinary}$$

- complementary / substituting goods

$$\frac{\partial q_i}{\partial p_j} = \frac{y(p_i p_j + p_i^2) - y p_j p_i}{(p_i p_j + p_i^2} = \frac{y p_i^2}{(p_i p_j + p_i^2)^2} > 0 \rightarrow \text{both goods are substitutes}$$

(b) Characterize each good with respect to the change of consumption resulting from income and price changes.

Problem 4 (*Optimal Consumption*)

Consider a utility-maximizing individual with a given income $y > 0$. The individual's utility function is given by

Cobb-Douglas Utility function
→ No cross-price effects
→ quantity is $\frac{\text{income}}{\text{own price}}$

$$U(q_1, q_2) = q_1^{\alpha} \cdot q_2^{\beta}, \quad \text{with} \quad \alpha > 0 \quad \text{and} \quad \beta > 0,$$

where $q_1 \geq 0$ and $q_2 \geq 0$ denote the quantities of good 1 and good 2, respectively. The goods prices are given by $p_1 > 0$ and $p_2 > 0$, respectively.

(a) Determine the individual demand function for each good.

Assume now that $\alpha = \beta = \frac{1}{2}$, $y = 600$ and $p_1 = 25$ and consider a price increase of good 2 from $p_2 = 25$ to $p_2 = 100$.

(b) Determine the optimal consumption bundles before and after the price increase and depict them in a diagram.

(c) Decompose the total effect of the price increase mathematically as well as diagrammatically into substitution and income effects.

(d) What income is necessary after the price increase, so that the individual can reach the same level of utility as before the price increase?

↳ often also in exams

Problem 5 (*Optimal Consumption*)

Consider a utility-maximizing individual with a given income $y = 100$. The individual's utility function is given by

$$U(q_1, q_2) = q_1 + 2q_2,$$

where $q_1 \geq 0$ and $q_2 \geq 0$ denote the quantities of good 1 and good 2, respectively. The goods prices are given by $p_1 = 4$ and $p_2 = 5$, respectively.

(A) If the marginal rate of substitution and the price ratio are never equal, no optimal consumption bundle can be determined.

Perfect substitutes – constant line "curve" of substitution

(B) The individual spends her entire budget on good 2. $\frac{4}{5} > \frac{1}{2}$

$\frac{1}{2} \neq \frac{4}{5}$
→ No interior solution, but "corner solution"
→ But consumption bundle can be determined

(C) The optimal consumption bundle contains twice as many units of good 2 as of good 1.

(D) The individual spends her entire budget on good 1.

a) $\max\limits_{q_1, q_2} U(q_1, q_2) = q_1^{\alpha} \cdot q_2^{\beta}$ s.t. $y \geq P_1 q_1 + P_2 q_2$

→ no more Lagrange; skip it

(i) $y = q_1 q_1 + P_2 q_2$

(ii) $\underbrace{\dfrac{\alpha q_1^{\alpha-1} q_2^{\beta}}{\beta q_1^{\alpha} q_2^{\beta-1}}}_{MRS} = \dfrac{P_1}{P_2} \iff \dfrac{\alpha}{\beta} = \dfrac{P_1}{P_2} \Rightarrow q_2 = \dfrac{\beta}{\alpha} \cdot \dfrac{P_1}{P_2} \cdot q_1$

$q_1 = \dfrac{\alpha}{\beta} \cdot \dfrac{P_2}{P_1} \cdot q_2$

(ii) in (i) $y = P_1 q_1 + \cancel{P_2} \dfrac{\beta}{\alpha} \dfrac{P_1}{\cancel{P_2}} q_1$

$= \dfrac{\alpha+\beta}{\alpha} \cdot P_1 q_1$

(iii) $\Rightarrow q_1 = \overbrace{\dfrac{\alpha}{\alpha+\beta} \cdot \dfrac{y}{P_1}}^{\text{Positive}} \rightarrow$ n ood

→ inary good

→ does not depend on P_2 → no cross-price influence

(iii) in (i) $y = P_1 \dfrac{\alpha}{\beta} \dfrac{P_2}{P_1} q_2 + P_2 q_2 = \dfrac{\alpha+\beta}{\beta} P_2 q_2$

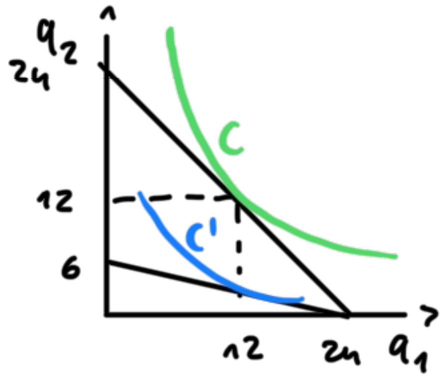
$- \alpha+\beta \quad 1_2$

b) $\max\limits_{q_1, q_2} U(q_1, q_2) = q_1^{\alpha} q_2^{\beta}$ s.t. $y \geq P_1 q_1 + P_2 q_2$

$\alpha = \beta = \frac{1}{2}$, $y = 600$, $P_1 = 25$

$\Rightarrow q_1 = 12$, $q_2 = \dfrac{300}{P_2}$

- $P_2 = 25 \Rightarrow q_2 = 12$
- $P_2 = 100 \Rightarrow q_2 = 3$



Problems 6-10 (*Optimal Consumption*)

Consider a utility-maximizing individual with a given income $y = 80$. The individual's utility function is given by

$$U(q_1, q_2) = q_1^{\frac{1}{2}} + q_2,$$

where $q_1 \geq 0$ and $q_2 \geq 0$ denote the quantities of good 1 and good 2, respectively. The price of good 1 is given by $p_1 = 1$. Consider a price decrease of good 2 from $p_2 = 8$ to $p_2 = 4$.

Problem 6

Before the price decrease, the individual's optimal consumption bundle is

(A) $q_1 = 8$ and $q_2 = 9$.

(B) $q_1 = 16$ and $q_2 = 8$.

(C) $q_1 = 24$ and $q_2 = 7$.

(D) $q_1 = 32$ and $q_2 = 6$.

$\max_{q_1, q_2} U(q_1, q_2) = q_1^{1/2} + q_2$ s.t. $y \geq p_1 q_1 + p_2 q_2$

(1) $y = \dots$

(2) $\underbrace{\frac{\partial U}{\partial q_1} = \frac{1}{2} q_1^{-\frac{1}{2}}}_{\text{MRS indep. of } q_2}$ $\rightarrow \frac{1}{2 q_1^{1/2}} = \frac{p_1}{p_2}$

MRS indep. of q_2 ✱
→ quasi-linear util. fn.

$q_1^{1/2} = \frac{1}{2} \frac{p_2}{p_1}$

$q_1 = \frac{1}{4} \left(\frac{p_2}{p_1}\right)^2$

✱ quasi-linear demand fn. is also indep. on y

(2) into (1) $y = p_1 \cdot \frac{1}{4} \left(\frac{p_2}{p_1}\right)^2 + p_2 q_2 \iff q_2 = \frac{y}{p_2} - \frac{1}{4} \frac{p_2}{p_1}$ ✱1

Problem 7

After the price decrease, the individual's optimal consumption bundle causes expenses of

(A) 24 for good 1.

(B) 40 for good 1.

(C) 60 for good 2.

(D) 76 for good 2.

$p_2 = 8$ $\quad q_1 = 16,\ q_2 = 8$

$p_2' = 4$ $\quad q_1 = 4,\ q_2 = 19$

Problem 8

Regarding good 1, the income effect of the price decrease amounts to

(A) $\text{IE}_1 = -4$.

(B) $\text{IE}_1 = 0$. No y in q_1 equation above

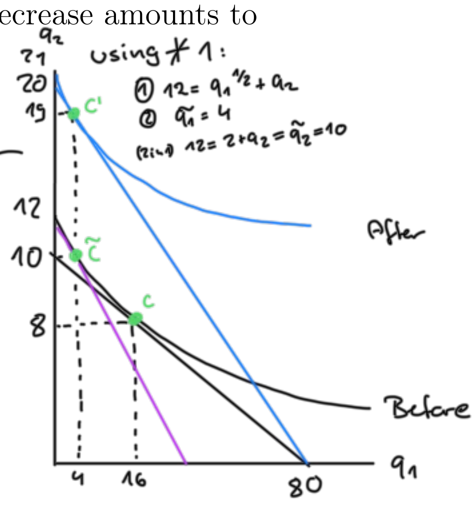
(C) $\text{IE}_1 = 4$.

(D) $\text{IE}_1 = 8$.

$IE_1 = 0$ → no income effect
$SE_1 = -12$

for q_2: $IE_2 = 9$ → normal good
$SE_2 = 2$

using ✱1:
① $12 = q_1^{1/2} + q_2$
② $\tilde{q}_1 = 4$
(2 in 1) $12 = 2 + q_2 = \tilde{q}_2 = 10$



Problem 9

Regarding good 2, the substitution effect of the price decrease amounts to

(A) $\text{SE}_2 = -4$.

(B) $\text{SE}_2 = -2$.

(C) $\text{SE}_2 = 2$.

(D) $\text{SE}_2 = 4$.

Problem 10

What income is necessary after the price decrease so that the individual can reach the same level of utility as before the price decrease?

(A) $Y = 36$.

(B) $Y = 40$.

(C) $Y = 44$.

(D) $Y = 48$.

$\tilde{Y} = 1 \cdot 4 + 4 \cdot 10 = 44$ (at $\tilde{c}$)

In addition to the Hicksian approach we learned, there's an alternative Slutskian approach. The lecture will always use the former. These influence the interpretation of the exact SE and IE values.