Exercise 3: Production and Supply

Problem 1 (*Production Function*)

Consider a firm with a production function

$$q = F(L, K) = L^{\alpha} K^{\beta}$$

with $\alpha \in (0, 1)$ and $\beta \in (0, 1)$, where q denotes output and L and K denote the inputs of labor and capital, respectively. Assume that $L > 0$ and $K > 0$.

(a) Show that the production function exhibits positive and decreasing marginal products of both inputs.

(b) Determine the conditions under which the production function exhibits constant, increasing, and decreasing returns to scale, respectively.

(c) Show that the isoquants of the production function are strictly convex.

Problem 2 (*Cost Minimization*)

Consider a firm with a production function

$$q = F(L, K) = (L \cdot K)^{\frac{1}{2}},$$

where $q \geq 0$ denotes output and $L \geq 0$ and $K \geq 0$ denote the inputs of labor and capital, respectively. Initially, the wage rate for labor is given by $w = 2.5$, and the rental rate for capital is given by $r = 2.5$. The firm produces an output of $q = 100$.

(a) Determine the cost-minimizing input bundle and depict it in a diagram.

Assume now that the wage rate for labor rises to $w = 10$, while the rental rate for capital remains at $r = 2.5$.

(b) Calculate and depict the effect of the increase in the wage rate

(i) on input costs if the same input bundle is employed as in (a),

(ii) on the cost-minimizing input bundle.

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a) $\frac{\partial F}{\partial L} = \alpha L^{\alpha-1} \cdot K^{\beta}$; $\frac{\partial F}{\partial K} = \beta \cdot L^{\alpha} \cdot K^{\beta-1}$ -> Both >0, no negative factor

$\frac{\partial^2 F}{\partial L^2} = (\alpha-1) \cdot \alpha \cdot L^{\alpha-2} \cdot K^{\beta}$; $\frac{\partial^2 F}{\partial K^2} = (\beta-1) \cdot \beta \cdot L^{a} \cdot K^{\beta-2}$ -> Both <0, $\alpha-1$ / $\beta-1$ is negative

Sidenote: Cross derivatives indicate add'l capital leads to add'l marginal value of work and vice-versa

$\frac{\partial^2 F}{\partial L \partial K} = \beta\alpha L^{\alpha-1} K^{\beta-1} > 0$; $\frac{\partial^2 F}{\partial K \partial L} = \alpha\beta \cdot L^{\alpha-1} \cdot K^{\beta-1} > 0$

b) $F(\lambda L, \lambda K) = (\lambda L)^{a} \cdot (\lambda K)^{\beta} = \lambda^{\alpha+\beta} \cdot L^{a} \cdot K^{\beta} = \lambda^{\alpha+\beta} \cdot F(L,K)$

I $\alpha+\beta = 1$ -> constant returns to scale

II $\alpha+\beta > 1$ -> increasing

III $\alpha+\beta < 1$ -> decreasing

Problems 3-5 (*Cost Minimization*)

Consider a firm with a production function

$$q = F(L, K) = (L \cdot K)^{\frac{1}{4}},$$

where $q \geq 0$ denotes output and $L \geq 0$ and $K \geq 0$ denote the inputs of labor and capital, respectively.

Problem 3

Provided that $L > 0$ and $K > 0$, a multiplication of both inputs by 4 implies a multiplication of output by

(A) 1.

(B) 2.

(C) 4.

(D) 16.

Problem 4

If the wage rate for labor is given by $w = 16$, and the rental rate for capital is given by $r = 4$, the firm's variable costs are

(A) $c(q) = 8q$.

(B) $c(q) = 12q$.

(C) $c(q) = 4q^2$.

(D) $c(q) = 16q^2$.

Problem 5

If the wage rate for labor is given by $w = 18$ and the rental rate for capital is given by $r = \frac{1}{2}$, the firm's marginal costs are

(A) $\text{MC}(q) = 4$.

(B) $\text{MC}(q) = 16$.

(C) $\text{MC}(q) = 8q$.

(D) $\text{MC}(q) = 12q$.

Problems 6-8 (*Profit Maximization*)

Consider a profit maximizing and price taking firm. Let $q \geq 0$ denote the firm's output, and let $p \geq 0$ denote the market price per unit of output. In the short run, the firm's total costs are

$$C(q) = 200 + 2q^2, \quad q \geq 0.$$

In the long run, the firm's total costs are

$$C(q) = \begin{cases} 200 + 2q^2, & q > 0 \\ 0, & q = 0. \end{cases}$$

Problem 6

For $q = 20$, marginal costs

(A) equal average total costs.

(B) are higher than average total costs.

(C) equal average variable costs.

(D) are lower than average variable costs.

Problem 7

If $p = 20$, the firm's supply

(A) is 0 in the short run as well as in the long run.

(B) is 5 in the short run and 0 in the long run.

(C) is 10 in the short run and 0 in the long run.

(D) is 20 in the short run as well as in the long run.

Problem 8

What is the threshold price, above which the firm's supply is $q > 0$ in the long run?

(A) $p = 10$

(B) $p = 20$

(C) $p = 30$

(D) $p = 40$