Exercise 7: Economic Growth

Problem 1 (*Steady State*)

Consider a closed economy in the long run. Output Y is determined by the production possibilities according to

$$Y = F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}},$$

where L denotes the labor force, and K denotes the capital stock. Output is used for consumption C and investment I. Investment equals savings sY, where $s \in (0, 1)$ denotes the saving rate. Savings are invested in the capital stock. In any period t, the labor force grows at the rate $n = -\frac{1}{20}$, while the capital stock depreciates at the rate $\delta = \frac{1}{10}$. Let lowercase letters denote quantities per worker.

(a) Calculate output per worker as a function of capital per worker.

(b) Calculate steady-state consumption per worker as a function of the saving rate.

(c) Calculate the golden-rule saving rate.

Problems 2-6 (*Steady State*)

Consider a closed economy in the long run. Output Y is determined by the production possibilities according to

$$Y = F(L, K) = L^{\frac{1}{3}} K^{\frac{2}{3}},$$

where L denotes the labor force, and K denotes the capital stock. Output is used for consumption C and investment I. Investment equals savings sY, where $s \in (0, 1)$ denotes the saving rate. Savings are invested in the capital stock. In any period t, the labor force grows at the rate $n = \frac{1}{6}$, while the capital stock depreciates at the rate $\delta = \frac{1}{6}$. Let lowercase letters denote quantities per worker.

Problem 2

In the steady state, output per worker is $y^* = 1$ if the saving rate is

(A) $s = \frac{1}{12}$.

(B) $s = \frac{1}{6}$.

(C) $s = \frac{1}{3}$.

(D) $s = \frac{2}{3}$.

Problem 3

If the saving rate is $s = \frac{1}{3}$, and capital per worker is $k = 1$,

(A) then output per worker y decreases over time.

(B) then output Y increases over time at a constant rate.

(C) then capital per worker k increases over time.

(D) then the capital stock K decreases over time at a constant rate.

Problem 4

The golden-rule saving rate is

(A) $s_{gold} = \frac{1}{12}$.

(B) $s_{gold} = \frac{1}{6}$.

(C) $s_{gold} = \frac{1}{3}$.

(D) $s_{gold} = \frac{2}{3}$.

Problem 5

In the golden-rule steady state, consumption per worker is

(A) $c^*_{gold} = \frac{16}{3}$.

(B) $c^*_{gold} = \frac{8}{3}$.

(C) $c^*_{gold} = \frac{4}{3}$.

(D) $c^*_{gold} = \frac{2}{3}$.

Problem 6

Any saving rate satisfying

(A) $s \in \left[0, \frac{1}{3}\right)$ implies a dynamically efficient steady state.

(B) $s \in \left(\frac{1}{3}, 1\right]$ implies a dynamically efficient steady state.

(C) $s \in \left[0, \frac{1}{3}\right)$ implies a dynamically inefficient steady state.

(D) $s \in \left(\frac{1}{3}, 1\right]$ implies a dynamically inefficient steady state.