Principles of Economics

Chapter 3: Production and Supply

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Agenda

Typically upward sloping line

3. Production and Supply
   - Cost Minimization
   - Profit Maximization
   - Individual Supply
   - Market Supply

Reading:

- Mankiw/Taylor (2023), Chapters 5, 6
- Varian (2014), Chapters 19-24

Model

Framework: Consider a representative firm. ← single good

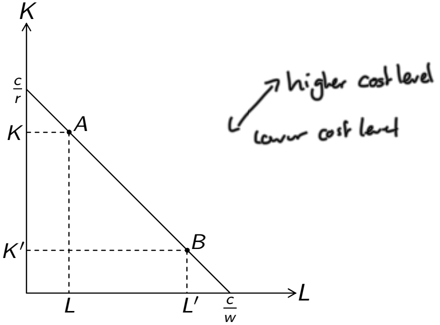
- The firm produces q units of a good (output) by employing two factors of production (inputs); L denotes the quantity of labor, while K denotes the quantity of capital.
- The firm is a price taker: It considers the output price p as well as the input prices w for labor and r for capital as given. (rental rate for $)
- The firm's input costs are $c = wL + rK$.
- The firm's revenue is $R(q) = pq$.

Input Costs

Isocost Line: Locus of all input bundles (L, K) that lead to the same level of input costs

No constraint, this is the objective:

$$K = \frac{c}{r} - \frac{w}{r} L$$

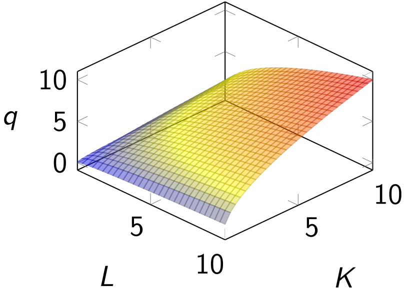


Input Price Ratio: Rate at which the firm can substitute one input for another at constant input costs

Production

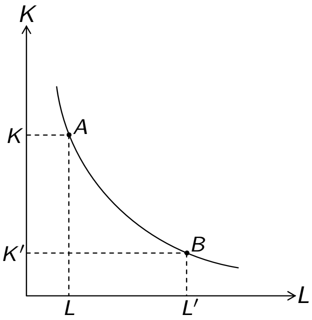
Production Function: The function $F(L, K)$ represents the firm's production technology; it expresses the maximum output q the firm can produce from any given input bundle (L, K).

Example: $q = F(L, K) = (L \cdot K)^{\frac{1}{2}}$



Production

Isoquant: Locus of all input bundles (L, K) that yield the same level of output $q = F(L, K)$



Marginal Rate of Technical Substitution: Rate at which the firm can substitute one input for another at constant output

$$\mathrm{MRTS}_{L,K} = \frac{\partial F / \partial L}{\partial F / \partial K}$$

Assumptions on Technology

Monotonicity: If input bundle A contains more of each input than input bundle B, then A yields more output than B. If input bundle A contains more of at least one input and not less of another, then A yields at least as much output as B. If in the latter case, A always yields more output than B, the technology is strictly monotonous.

- If the technology is strictly monotonous, then isoquants are negatively sloped.

Convexity: If input bundles A and B yield the same output, then any convex combination of A and B yields at least as much output as A or B. If any strictly convex combination of A and B yields more output than A or B, the technology is strictly convex.

- If the technology is (strictly) convex, then isoquants are (strictly) convex.

Extreme Cases of Technology

Perfect Substitutes: Two inputs that can be substituted for one another at a constant rate while output remains constant

- Linear isoquants

Perfect Complements: Two inputs that should be employed in fixed proportions

- Orthogonal isoquants

Scale of Production

Returns to Scale: If all inputs are multiplied by a constant λ, the resulting change in output can be proportional, more than proportional, or less than proportional. The production function $F(L, K)$ exhibits

- constant returns to scale if

$$F(\lambda L, \lambda K) = \lambda F(L, K) \quad \text{for any} \quad \lambda > 0,$$

- increasing returns to scale if

$$F(\lambda L, \lambda K) > \lambda F(L, K) \quad \text{for any} \quad \lambda > 1,$$

- decreasing returns to scale if

$$F(\lambda L, \lambda K) < \lambda F(L, K) \quad \text{for any} \quad \lambda > 1.$$

Cost Minimum

Optimization Problem: The firm minimizes input costs with respect to input employment subject to a given output.

$$\min_{L,K} \quad c = wL + rK \quad \text{s.t.} \quad q = F(L, K)$$

Any interior solution of the minimization problem must satisfy the following conditions:

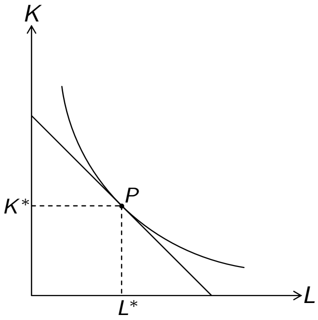
$$q = F(L, K),$$

$$\mathrm{MRTS}_{L,K} = \frac{\partial F/\partial L}{\partial F/\partial K} = \frac{w}{r}.$$

Cost Minimum

Interior Solution: The rate at which the firm can substitute labor for capital at constant output must equal the rate at which it can substitute labor for capital at constant input costs.

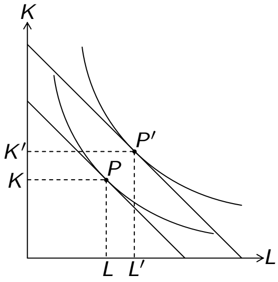
- In the optimal input bundle, the slope of the isoquant equals the slope of the isocost line.



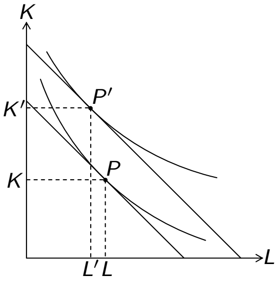
Cost Minimum

Change in Output: An increase (a decrease) in output causes higher (lower) input costs; $\frac{\partial c}{\partial q} > 0$.

- In order to increase output, more of at least one input must be employed.



More of both inputs



More Capital, less Labor

Production Costs

Total Costs: Sum of fixed and variable costs

- Fixed costs c^f are independent of output.
- Variable costs $c(q)$ represent minimum input costs as a function of output.

$$C(q) = c^f + c(q)$$

Average Costs: Costs per unit of output

- Let $AC(q)$ denote average total costs, and let $ac(q)$ denote average variable costs.

$$AC(q) = \frac{C(q)}{q} = \frac{c^f}{q} + \underbrace{\frac{c(q)}{q}}_{ac(q)}$$

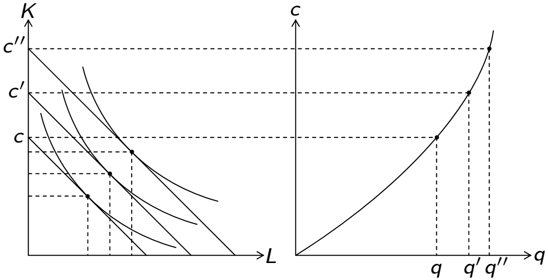
Marginal Costs: Change in total costs resulting from a marginal increase in output

$$MC(q) = \frac{dC(q)}{dq}$$

Production Costs

Variable Cost Curve: Variable costs increase (decrease), as output increases (decreases); $\frac{dc(q)}{dq} > 0$

- The price of capital is normalized to $r = 1$.



Cost Minimization & Variable Costs

Production Costs

Short-Run Total Costs: In the short run, fixed costs are sunk costs.

- Sunk Costs: Incurred costs that cannot be recovered

$$C(q) = c^f + c(q), \quad q \geq 0$$

Long-Run Total Costs: In the long run, non-variable costs must be quasi-fixed costs.

- Quasi-Fixed Costs: Costs that arise if the firm starts production but do not vary as output increases

$$C(q) = \begin{cases} c^f + c(q), & q > 0 \\ 0, & q = 0 \end{cases}$$

Profit Maximum

Optimization Problem: The firm maximizes profit with respect to output.

- Profit $\pi(q)$ equals revenue $R(q)$ minus total costs $C(q)$.

$$\max_{q} \quad \pi(q) = R(q) - C(q)$$

In any interior solution, marginal revenue $MR(q)$ equals marginal costs $MC(q)$.

$$\underbrace{\frac{dR(q)}{dq}}_{MR(q)} = \underbrace{\frac{dC(q)}{dq}}_{MC(q)}.$$

Marginal revenue is the change in revenue resulting from a marginal increase in output.

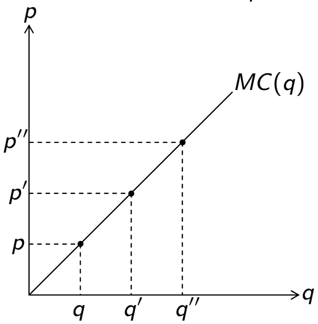
- For a price-taking firm, marginal revenue equals the output price so that an interior solution requires

$$p = MC(q).$$

Profit Maximum

Marginal Cost Curve: Assume that costs are strictly convex. Hence, marginal costs increase (decrease) as output increases (decreases); $\frac{dMC(q)}{dq} > 0$.

- Interior Solution: An increase (a decrease) in the output price results in an increase (a decrease) of the profit-maximizing output; $\frac{\partial q}{\partial p} > 0$.



Profit-Maximizing Output

Individual Supply Curve

Optimal Production: The firm supplies either the output $q > 0$ satisfying the condition $p = MC(q)$ or the output $q = 0$.

- In the short run, the firm supplies $q > 0$ if and only if

$$R(q) \geq c(q) \quad \Leftrightarrow \quad p \geq ac(q).$$

- In the long run, the firm supplies $q > 0$ if and only if

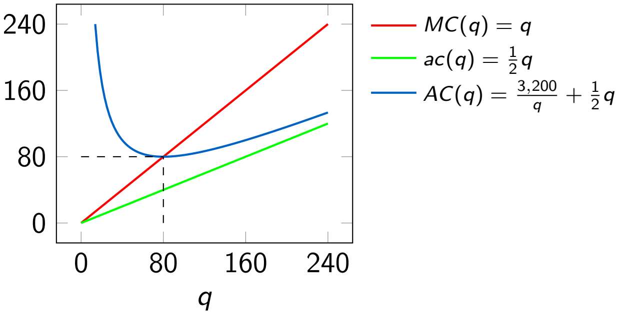
$$R(q) \geq C(q) \quad \Leftrightarrow \quad p \geq AC(q).$$

Individual Supply Curve: The short-run (long-run) individual supply curve corresponds to the segment of the marginal cost curve that runs above the average variable (average total) cost curve.

Individual Supply Curve

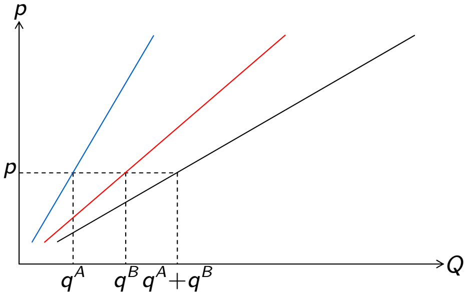
Example: Consider a firm with total costs $C(q) = 3{,}200 + \frac{1}{2}q^2$.

- Short-run individual supply: $q = p$
- Long-run individual supply: $q = \begin{cases} p, & p \geq 80 \\ 0, & p < 80 \end{cases}$



Market Supply Curve

Market Supply: Sum of individual supply quantities of a good; $Q = \sum q$



Individual & Market Supply Curves

Law of Supply: Empirical observation that, ceteris paribus, the market supply of a good increases (decreases) as its price increases (decreases); $\frac{\partial Q}{\partial p} > 0$