Principles of Economics

Chapter 7: Economic Growth

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Agenda

7. Economic Growth
   - Steady State
   - Optimal Capital Accumulation
   - Technological Progress

Reading:

- Mankiw/Taylor (2023), Chapter 21
- Mankiw (2025), Chapters 9, 10

Model

Framework: Consider a closed economy in the long run where all input and output prices are flexible.

- Output Y is determined by the production possibilities; the production function and the supply of inputs, i.e. the labor force L and the capital stock K.

$$Y = F(L, K)$$

- Output is used for consumption C and investment I. Investment equals savings sY, where $s \in [0, 1]$ denotes the saving rate.

$$Y = C + sY$$

- Savings are invested in the capital stock.

Production Function

Properties: The production function satisfies the following conditions.

- Constant returns to scale:

$$F(\lambda L, \lambda K) = \lambda F(L, K) \quad \text{for any} \quad \lambda > 0$$

- Positive and decreasing marginal products:

$$\frac{\partial F}{\partial L} > 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial L^2} < 0$$

$$\frac{\partial F}{\partial K} > 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial K^2} < 0$$

- Inada-Conditions:

$$\lim_{L \to 0} \left(\frac{\partial F}{\partial L} \right) = \infty \quad \text{and} \quad \lim_{L \to \infty} \left(\frac{\partial F}{\partial L} \right) = 0$$

$$\lim_{K \to 0} \left(\frac{\partial F}{\partial K} \right) = \infty \quad \text{and} \quad \lim_{K \to \infty} \left(\frac{\partial F}{\partial K} \right) = 0$$

Production Function

Intensive Form: Denote all quantities in per-worker terms, using lowercase letters.

- Constant returns to scale imply:

$$y = f(k)$$

- Positive and decreasing marginal product:

$$f'(k) > 0 \quad \text{and} \quad f''(k) < 0$$

- Inada-Conditions:

$$\lim_{k \to 0} f'(k) = \infty \quad \text{and} \quad \lim_{k \to \infty} f'(k) = 0$$

Supply of Inputs

Population Growth: In any period t, the labor force grows at a constant rate n.

$$L_{t+1} = (1 + n)L_t$$

Capital Accumulation: In any period t, a fraction s of output is invested in the capital stock, while capital depreciates at the rate $\delta \in [0, 1]$.

$$K_{t+1} = K_t + sY_t - \delta K_t$$

Ceteris paribus, capital per worker increases with savings and decreases with depreciation and population growth.

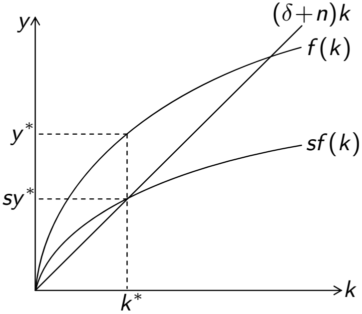
$$(1 + n)k_{t+1} = k_t + sf(k_t) - \delta k_t$$

Capital per Worker in the Steady State

Steady State: Situation in which capital per worker is constant over time, $k_t = k_{t+1} = k^*$

- In a steady state, savings are equal to break-even investment, i.e. investment per worker exactly offsets the decrease in capital per worker due to depreciation and population growth.

$$sf(k^*) = (\delta + n)k^*$$



Capital per Worker in the Steady State

Convergence: If capital per worker in period t

- is below the steady-state level, $k_t < k^*$, then savings exceed break-even investment, $sf(k_t) > (\delta + n)k_t$, and capital per worker increases.
- is above the steady-state level, $k_t > k^*$, then savings fall short of the break-even investment, $sf(k_t) < (\delta + n)k_t$, and capital per worker decreases.

Parameter Changes: Ceteris paribus, an increase in

- the saving rate s implies an increase in steady-state capital per worker.
- the depreciation rate δ implies a decrease in steady-state capital per worker.
- the rate of population growth n implies a decrease in steady-state capital per worker.

Golden Rule

Optimization Problem: Maximize consumption per worker in the steady state.

- In any period t, consumption per worker is

$$c_t = (1 - s)y_t = f(k_t) - sf(k_t).$$

- Using the steady-state condition yields the objective function:

$$\max_{k^*} \quad c^* = f(k^*) - (\delta + n)k^*$$

- Hence, the necessary condition for maximum consumption per worker in the steady state, i.e. the golden rule of capital accumulation is

$$f'(k^*_{gold}) = \delta + n.$$

- The corresponding saving rate, which maximizes consumption per worker in the steady state, is s_{gold}.

Golden Rule

Dynamic Inefficiency: No trade-off between present and future consumption

- If the saving rate exceeds its golden-rule level, $s > s_{gold}$, then a decrease in the saving rate implies an immediate increase in consumption as well as an increase in steady-state consumption per worker.

Dynamic Efficiency: Trade-off between present and future consumption

- If the saving rate falls short of its golden-rule level, $s < s_{gold}$, then an increase in the saving rate implies an immediate decrease in consumption but an increase in steady-state consumption per worker.

Model Extension

Production Function: Output is a function of the effective labor force $A \cdot L$ and the capital stock K.

- The effective labor force takes into account the number of workers L and workers' productivity A.

$$Y = F(A \cdot L, K)$$

Labor Productivity Growth: In any period t, workers' productivity grows at the rate g.

- In the steady state, the capital stock per worker also grows at the rate g.