Solution 1: Specialization and Trade

Problem 1 (*Gains from Trade*)

(a) Jack can produce more fish and more rum per hour than Will. Thus, Jack can produce each of the two goods using less resources (hours) than Will.

kg per hour		
	Fish	Rum
Jack	1	1/4
Will	1/5	1/5

hours per kg		
	Fish	Rum
Jack	1	4
Will	5	5

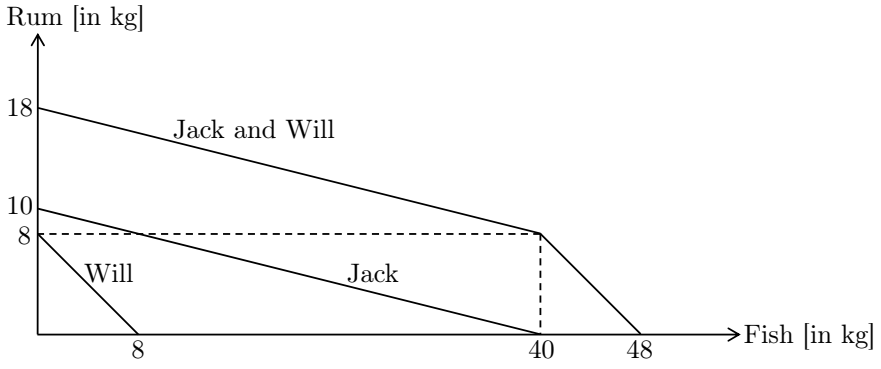
$\Rightarrow$ Jack has absolute advantages in the production of both goods.

Jack can produce fish at lower opportunity costs than Will. Consequently, Will can produce Rum at lower opportunity costs than Jack.

Opportunity Costs per kg		
	Fish [in kg Rum]	Rum [in kg Fish]
Jack	1/4	4
Will	1	1

$\Rightarrow$ Jack has a comparative advantage in the production of fish, while Will has a comparative advantage in the production of rum.

(b) Transformation Curves:



Transformation Curves

The axis intercepts and the corner point of the joint transformation curve can be determined as follows:

⇒ If both, Jack and Will, produce only fish, then the overall production is 48 kg fish and 0 kg rum.

⇒ If both, Jack and Will, produce only rum, then the overall production is 0 kg fish and 18 kg rum.

⇒ If Jack produces only fish and Will produces only rum, i.e. if both specialize completely according to their respective comparative advantage, the overall production is 40 kg fish and 8 kg rum.

(c) Under autarky, individual consumption equals individual production. In this case, each pirate produces 5 kg rum per week and spends the remaining time producing fish. This results in 20 kg fish produced by Jack and 3 kg fish produced by Will.
Under specialization and trade, individual consumption can differ from individual production. Will has a comparative advantage in the production of rum, of which he can maximally produce 8 kg. Since 10 kg rum are required for both pirates together, Will should produce only rum. The remaining 2 kg rum must be produced by Jack, even though Jack's comparative advantage lies in the production of fish. In the remaining time, Jack can produce 32 kg fish. If 1 kg rum is traded for 3 kg fish, Jack can consume 23 kg fish, while Will can consume 9 kg fish in addition to the 5 kg rum that each of them consumes.

Production & Consumption

	Fish	Rum
Jack	20	5
Will	3	5

Autarky

Production (Consumption)

	Fish	Rum
Jack	32 (23)	2 (5)
Will	0 (9)	8 (5)

Specialization & Trade

⇒ Specialization and trade is beneficial for both pirates.

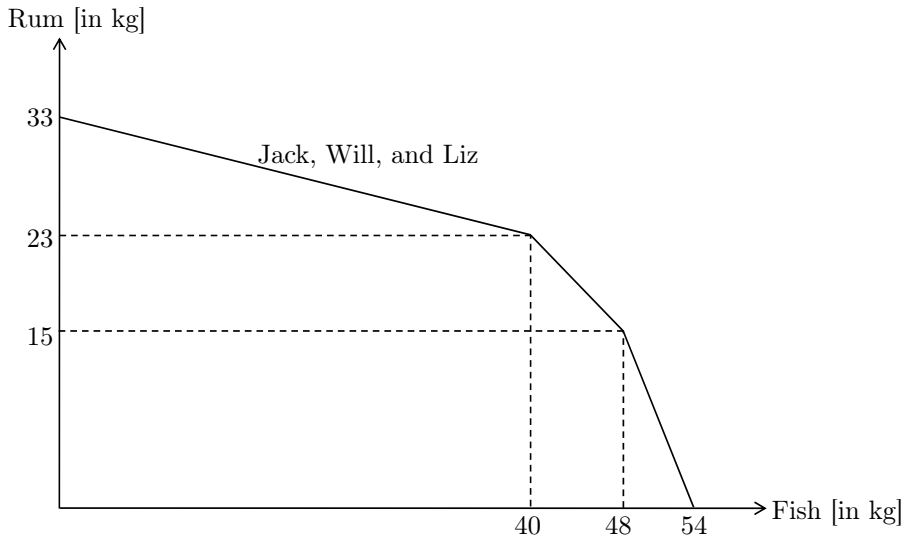
(d) Liz joins Jack and Will.

kg per hour		
	Fish	Rum
Jack	1	1/4
Will	1/5	1/5
Liz	1/5	1/2

hours per kg		
	Fish	Rum
Jack	1	4
Will	5	5
Liz	5	2

Opportunity Costs per kg		
	Fish [in kg Rum]	Rum [in kg Fish]
Jack	1/4	4
Will	1	1
Liz	5/2	2/5

Joint transformation curve



Transformation Curve

The axis intercepts and corner points can be determined as follows:

⇒ If Jack, Will, and Liz produce only fish, then the overall production is 54 kg fish and 0 kg rum.

⇒ If Jack, Will, and Liz produce only rum, then the overall production is 0 kg fish and 33 kg rum.

⇒ If Jack produces only fish and Liz produces only rum, i.e. both completely specialize according to their comparative advantages, and if Will produces only rum, then the overall production is 40 kg fish and 23 kg rum.

⇒ If Jack produces only fish and Liz produces only rum, i.e. both completely specialize according to their comparative advantages, and if Will produces only fish, then the overall production is 48 kg fish and 15 kg rum.

(e) Under autarky, individual consumption equals individual production. In this case, each pirate produces 5 kg rum per week and spends the remaining time producing fish. This results in 20 kg fish produced by Jack, 3 kg fish produced by Will, and 4 kg fish produced by Liz.
Under specialization and trade, individual consumption can differ from individual production. Liz has a comparative advantage over Jack and Will in the production of rum, of which she can maximally produce 15 kg. Since 15 kg rum are required for all three pirates together, Liz should produce only rum. Then Jack and Will, who both have a comparative advantage over Liz in the production of fish, should produce only fish. This results in the production of 48 kg fish. If 1 kg rum is traded for $\frac{4}{5}$ kg fish, Jack can consume 36 kg fish, Will can consume 4 kg fish, and Liz can consume 8 kg fish in addition to the 5 kg rum that each of them consumes.

Production & Consumption

	Fish	Rum
Jack	20	5
Will	3	5
Liz	4	5

Autarky

Production (Consumption)

	Fish	Rum
Jack	40 (36)	0 (5)
Will	8 (4)	0 (5)
Liz	0 (8)	15 (5)

Specialization & Trade

⇒ Specialization and trade is beneficial for all three pirates.

Problems 2-6 (*Gains from Trade*)

Days per unit		
	Car Bodies	Engines
Carl	16	4
Gottlieb	10	5

Opportunity Costs per unit		
	Car Bodies [in units of Engines]	Engines [in units of Car Bodies]
Carl	4	1/4
Gottlieb	2	1/2

Let B denote the quantity of car bodies (in units) and let E denote the quantity of engines (in units). Then Carl's time constraint is

$$300 = 16B + 4E.$$

Rearranging yields Carl's individual transformation curve:

$$B = 18.75 - \frac{1}{4}E. \tag{1}$$

Gottlieb's time constraint is

$$300 = 10B + 5E.$$

Rearranging yields Gottlieb's individual transformation curve:

$$B = 30 - \frac{1}{2}E. \tag{2}$$

Then, the joint transformation curve of Carl and Gottlieb is given by the equation

$$B = \begin{cases} 48.75 - \frac{1}{4}E & 0 \leq E \leq 75 \\ 67.5 - \frac{1}{2}E & 75 \leq E \leq 135. \end{cases} \tag{3}$$

Problem 2

Gottlieb has both, an absolute and a comparative advantage in the production of car bodies.

$\Rightarrow$ **(B)** is correct.

Problem 3

To manufacture one vehicle, Carl needs $16 + 4 = 20$ days, while Gottlieb needs $10 + 5 = 15$ days. Under autarky, Carl can maximally manufacture $300 \div 20 = 15$ vehicles per year, while Gottlieb can maximally manufacture $300 \div 15 = 20$ vehicles per year.

These results can also be derived by substituting the condition $B = E$ into equations (1) and (2), respectively.

$\Rightarrow$ **(C)** is correct.

Problem 4

Gottlieb should completely specialize according to his comparative advantage, and thus manufacture $300 \div 10 = 30$ car bodies. If Carl also completely specialized according to his comparative advantage, he would manufacture $300 \div 4 = 75$ engines. In this case, only 30 vehicles would be produced, with 45 engines remaining unused. Consequently, Carl should produce both, car bodies and engines. To begin with, he should produce 30 engines for Gottlieb's 30 car bodies to make 30 vehicles, for which he needs $30 \cdot 4 = 120$ days. In the remaining 180 days, he should produce engines and car bodies, resulting in $180 \div 20 = 9$ complete vehicles. Thus, if Carl and Gottlieb cooperate, they can maximally produce $30 + 9 = 39$ vehicles per year.

This result can also be derived by substituting the condition $B = E$ into equation (3).

$\Rightarrow$ **(B)** is correct.

Problem 5

Carl and Gottlieb can realize mutual gains from trade if they agree on terms of trade lying in between their respective opportunity costs, i.e. in between $\frac{1}{4}$ and $\frac{1}{2}$ car bodies per engine, or equivalently, in between 2 and 4 engines per car body.

$\Rightarrow$ **(A)** is correct.

Problem 6

If they cooperate, Carl and Gottlieb can maximally produce 39 vehicles per year (see also problem 4), implying that the combination 39 car bodies and 39 engines is located on the two engineers' joint transformation curve. Consequently, the combination 39 car bodies and 50 engines is *not* located on their joint transformation curve but beyond it.

The fact that the other three combinations of car bodies and engines are located on the two engineers' joint transformation curve can be verified by substituting the respective quantitites into equation (3).

$\Rightarrow$ **(D)** is correct.