Solution 2: Consumption and Demand

Problem 1 (*Budget Constraint*)

(a) The individual's budget constraint in terms of time is $Z = L + F$, while her budget constraint in terms of earned income is $wL = pq$. Solving the former for L and substituting it into the latter yields $w(Z - F) = pq$. It follows that her budget constraint in terms of potential income is

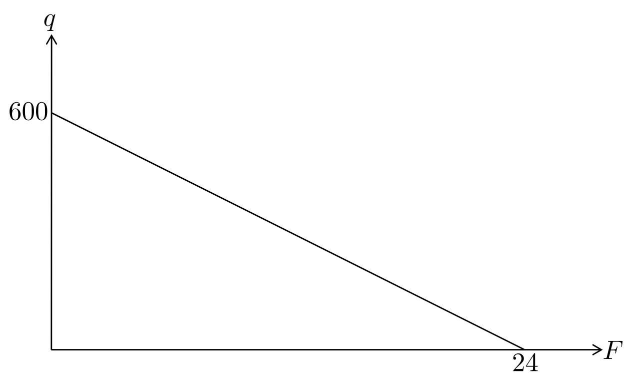
$$wZ = pq + wF.$$

Solving for q yields the budget line

$$q = \frac{wZ}{p} - \frac{w}{p}F.$$

Substituting $Z = 24$, $w = 25$, and $p = 1$ gives

$$q = 600 - 25F.$$



Budget line

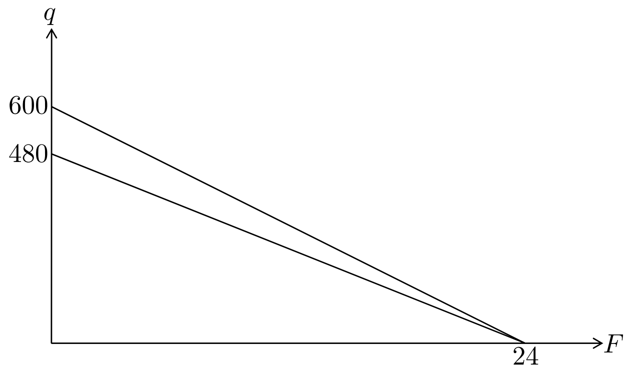
(b) The individual's budget line

(i) if an income tax reduces the wage rate to $(1-t)w = 20$ is

$$q = 480 - 20F,$$

(ii) if a consumption tax raises the price of the consumption good to $(1+\tau)p = 1.25$ is

$$q = 480 - 20F,$$



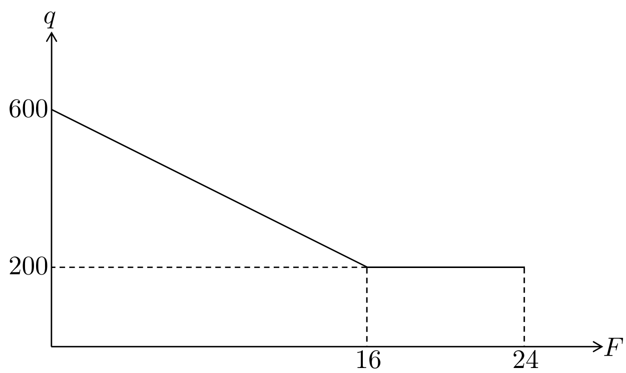
Budget line before and after taxes

(iii) if a social transfer

$$S = \begin{cases} 0, & wL \geq 200 \\ 200 - wL, & wL < 200 \end{cases}$$

subsidizes earned incomes is

$$q = \begin{cases} 600 - 25F, & F \leq 16 \\ 200, & F > 16. \end{cases}$$



Budget line with social transfer

Problem 2 (*Assumptions on Preferences*)

The individual's preferences regarding the consumption bundles $A = (8, 2)$, $B = (2, 8)$, and $C = (6, 6)$ involve the following relations: $A \sim B$ and $B \succ C$.[1]

- If her preferences are **complete**, she can compare the consumption bundles A and C such that $A \sim C$ or $A \succ C$ or $C \succ A$.
- If her preferences are **complete** and **transitive**, then it follows from $A \sim B$ and $B \succ C$ that $A \succ C$.
- If her preferences are **complete**, **transitive**, and **convex**, they cannot be **monotonous**. To see this, consider a consumption bundle D that is a convex combination of the consumption bundles A and B. By definition,

  $$D = \lambda A + (1 - \lambda)B \succeq B \quad \text{for any} \quad \lambda \in [0, 1].$$

  Since $B \succ C$, it follows that

  $$D = \lambda A + (1 - \lambda)B \succ C \quad \text{for any} \quad \lambda \in [0, 1].$$

  For $\lambda = \frac{1}{2}$, this implies $D = (5, 5) \succ C = (6, 6)$ which violates monotonicity (because it would imply that „less is better").

Thus, the four assumptions cannot hold together in this case.

[1]The symbol $\sim$ represents the indifference relation, i.e. the individual is indifferent between A and B. The symbol $\succ$ represents the strong preference relation, i.e. the individual strictly prefers B to C.

Problem 3 (*Individual Demand*)

(a) Individual demand for a particular good follows from utility maximization.

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must be satisfied with equality.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve running through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{1}{2} q_1^{-\frac{1}{2}}}{\frac{1}{2} q_2^{-\frac{1}{2}}}}_{\text{MRS}_{1,2}} = \frac{p_1}{p_2} \Leftrightarrow \left(\frac{q_2}{q_1}\right)^{\frac{1}{2}} = \frac{p_1}{p_2} \Rightarrow q_2 = \left(\frac{p_1}{p_2}\right)^2 q_1 \Leftrightarrow q_1 = \left(\frac{p_2}{p_1}\right)^2 q_2$$

Substituting (ii) into (i) yields

$$q_1 = \frac{y p_2}{p_1 p_2 + p_1^2}, \quad q_2 = \frac{y p_1}{p_1 p_2 + p_2^2}.$$

Thus, the individual demand for good $i \in \{1, 2\}$ is

$$q_i(p_i, p_j, y) = \frac{y p_j}{p_i p_j + p_i^2},$$

where $j \in \{1, 2\}$ and $j \neq i$.

(b) The demand for good $i \in \{1, 2\}$ increases (decreases) as income increases (decreases).

$$\frac{\partial q_i(p_i, p_j, y)}{\partial y} = \frac{p_j}{p_i p_j + p_i^2} > 0$$

$\Rightarrow$ Both goods are normal goods.

The demand for good $i \in \{1, 2\}$ decreases (increases) as the respective price p_i increases (decreases).

$$\frac{\partial q_i(p_i, p_j, y)}{\partial p_i} = -\frac{y p_j (p_j + 2 p_i)}{\left(p_i p_j + p_i^2\right)^2} < 0$$

$\Rightarrow$ Both goods are ordinary goods.

The demand for good $i \in \{1,2\}$ increases (decreases) as the price p_j of the other good increases (decreases).

$$\frac{\partial q_i(p_i, p_j, y)}{\partial p_j} = \frac{y p_i^2}{(p_i p_j + p_i^2)^2} > 0$$

$\Rightarrow$ The goods are substitutes.

Problem 4 (*Substitution and Income Effects*)

$U(q_1, q_2) = q_1^{\alpha} \cdot q_2^{\beta}$ is a Cobb-Douglas utility function.[2]

(a) Optimal consumption

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must be satisfied with equality.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve running through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\alpha q_1^{\alpha-1} \cdot q_2^{\beta}}{\beta q_1^{\alpha} \cdot q_2^{\beta-1}}}_{\text{MRS}_{1,2}} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad \frac{\alpha}{\beta} \cdot \frac{q_2}{q_1} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad q_2 = \frac{\beta}{\alpha} \cdot \frac{p_1}{p_2} q_1 \quad \Leftrightarrow \quad q_1 = \frac{\alpha}{\beta} \cdot \frac{p_2}{p_1} q_2$$

Substituting (ii) into (i) yields

$$q_1 = \frac{\alpha}{\alpha + \beta} \cdot \frac{y}{p_1}, \quad q_2 = \frac{\beta}{\alpha + \beta} \cdot \frac{y}{p_2}.$$

Note the following properties of Cobb-Douglas utility functions:

1. The demand for each of the two goods is independent of the price of the other good. Hence, the goods are neither substitutes nor complements.
2. Constant fractions of income are spent on the two goods. In any optimum, the individual spends the fraction $\frac{\alpha}{\alpha+\beta}$ of her income on good 1 and the fraction $\frac{\beta}{\alpha+\beta}$ of her income on good 2.

[2]This class of functions is named after Charles Cobb and Paul Douglas.

Substituting $\alpha = \beta = \frac{1}{2}$, $y = 600$ and $p_1 = 25$ yields

$$q_1 = 12, \quad q_2 = \frac{300}{p_2}.$$

(b) If $p_2 = 25$, the optimal consumption bundle C is

$$q_1 = 12, \quad q_2 = 12.$$

If $p_2' = 100$, the optimal consumption bundle C' is

$$q_1' = 12, \quad q_2' = 3.$$

(c) The total effect of the price change $C \to C'$ can be decomposed into the substitution effect $C \to \tilde{C}$ and the income effect $\tilde{C} \to C'$, where $\tilde{C}$ is a hypothetical consumption bundle. Given the new price ratio $\frac{p_1}{p_2'} = \frac{1}{4}$, the individual would choose $\tilde{C}$ if her income was compensated to the extent that she could obtain the initial level of utility $(12 \cdot 12)^{\frac{1}{2}} = 12$.

(i) $\tilde{C}$ must be located on the initial indifference curve through C.

$$(q_1 \cdot q_2)^{\frac{1}{2}} = 12$$

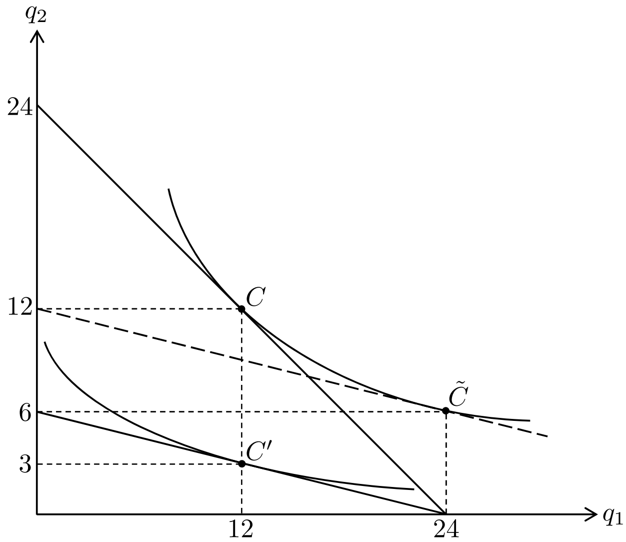
(ii) $\tilde{C}$ must be located where the initial indifference curve is tangent to the hypothetical budget line parallel to the new budget line through C'.

$$q_2 = \frac{1}{4} q_1 \quad \Leftrightarrow \quad q_1 = 4q_2$$

Substituting (ii) into (i) yields

$$\tilde{q}_1 = 24, \quad \tilde{q}_2 = 6.$$

It follows that for good 1, the substitution effect is $\text{SE}_1 = 12$, and the income effect is $\text{IE}_1 = -12$ while for good 2, the substitution effect is $\text{SE}_2 = -6$, and the income effect is $\text{IE}_2 = -3$.



Substitution- and Income Effect

(d) The hypothetical income necessary after the price increase so that the individual can afford the hypothetical consumption bundle $\tilde{C}$ and therefore obtain the initial level of utility is

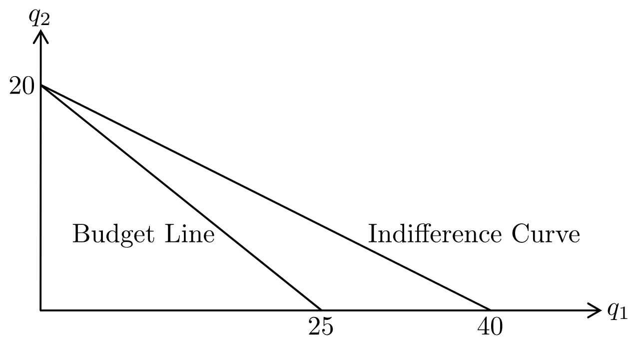
$$\tilde{y} = p_1\tilde{q}_1 + p_2'\tilde{q}_2 = 25 \cdot 24 + 100 \cdot 6 = 1{,}200.$$

Problem 5 (*Optimal Consumption*)

The two goods are perfect substitutes for the individual, because the marginal rate of substitution is constant. The fact that

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}_{1,2}} = \frac{1}{2} < \frac{4}{5} = \frac{p_1}{p_2}$$

implies that the (linear) indifference curves are always flatter than the budget line, so that the individual's optimization problem has no interior solution.



Corner Solution

The optimal consumption bundle is $q_1 = 0$ and $q_2 = 20$. Thus, the individual spends her entire budget on good 2.

$\Rightarrow$ **(B)** is correct.

Problem 6-10 (*Optimal Consumption*)

$U(q_1, q_2) = q_1^{\frac{1}{2}} + q_2$ is a quasi-linear utility function.[3]

Optimal consumption

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must be satisfied with equality.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{1}{2} q_1^{-\frac{1}{2}}}_{\text{MRS}_{1,2}} = \frac{p_1}{p_2} \quad \Rightarrow \quad q_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2$$

Substituting (ii) into (i) yields

$$q_2 = \frac{y}{p_2} - \frac{p_2}{4p_1}.$$

[3]Quasi-linear utiltiy functions are linear in one argument.

Substituting $y = 80$ and $p_1 = 1$ yields

$$q_1 = \frac{(p_2)^2}{4} \quad \text{and} \quad q_2 = \frac{80}{p_2} - \frac{p_2}{4}.$$

Note the following properties of quasi-linear utility functions:

1. The MRS is independent of the good that enters the utility function linearly.
2. The demand for the good that enters the utility function non-linearly is independent of income (given that the utility maximization problem has an interior solution).

Problem 6

If $p_2 = 8$, the individual's optimal consumption bundle is $q_1 = 16$ and $q_2 = 8$.

$\Rightarrow$ **(B)** is correct.

Problem 7

If $p_2 = 4$, the individual's optimal consumption bundle is $q_1 = 4$ and $q_2 = 19$. Thus, the optimal consumption bundle causes expenses of $4 \cdot 19 = 76$ for good 2 (and 4 for good 1).

$\Rightarrow$ **(D)** is correct.

The total effect of the price change $C \to C'$ can be decomposed into the substitution effect $C \to \tilde{C}$ and the income effect $\tilde{C} \to C'$, where $\tilde{C}$ is a hypothetical consumption bundle. Given the new price ratio $\frac{p_1}{p_2'} = \frac{1}{4}$, the individual would choose $\tilde{C}$ if her income was compensated to the extent that she could obtain the initial level of utility $U = 16^{\frac{1}{2}} + 8 = 12$.

(i) $\tilde{C}$ must be located on the initial indifference curve through C.

$$q_1^{\frac{1}{2}} + q_2 = 12.$$

(ii) $\tilde{C}$ must be located where the initial indifference curve is tangent to the hypothetical budget line parallel to the new budget line through C'.

$$q_1 = 4$$

Substituting (ii) into (i) yields

$$\tilde{q}_1 = 4, \quad \tilde{q}_2 = 10.$$

Problem 8

Regarding good 1, the total effect of the price decrease amounts to -12, of which the substutution effect is -12 and the income effect is 0.

$\Rightarrow$ **(B)** is correct.

Problem 9

Regarding good 2, the total effect of the price decrease amounts to $+11$, of which the substitution effect is $+2$ and the income effect is $+9$.

$\Rightarrow$ **(C)** is correct.

Problem 10

In order to afford the hypothetical consumption bundle, the individual needs a hypothetical income of $1 \cdot 4 + 4 \cdot 10 = 44$.

$\Rightarrow$ **(C)** is correct.