Solution 4: Perfect Competition

Problem 1 (*Competitive Equilibrium*)

(a) If a firm produces $q > 0$, this quantity must satisfy the condition for a profit maximum $p = \text{MC}(q)$, which implies

$$p = 2q \quad \Leftrightarrow \quad q = \frac{1}{2}p.$$

In the long run, a firm will produce $q > 0$ according to the above condition if and only if this yields a non-negative profit. Otherwise, the firm will produce $q = 0$. The firm's profit is zero at the break-even quantity, where marginal costs equal average total costs.

$$\text{MC}(q) = 2q = \frac{c^f}{q} + q = \text{AC}(q) \quad \Rightarrow \quad q = \sqrt{c^f}$$

The corresponding threshold price, which induces the firm to produce the break-even quantity, is

$$p = \text{MC}\left(\sqrt{c^f}\right) \quad \Rightarrow \quad p = 2\sqrt{c^f}.$$

It follows that individual supply is

$$q(p) = \begin{cases} \frac{1}{2}p, & p \geq 2\sqrt{c^f} \\ 0, & p < 2\sqrt{c^f}, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} \frac{n}{2}p, & p \geq 2\sqrt{c^f} \\ 0, & p < 2\sqrt{c^f}. \end{cases}$$

In equilibrium, market demand equals market supply:

$$Q^D(p) = a - p = \frac{n}{2}p = Q^S(p).$$

The number of firms that leads to zero profits for each firm equalizes market demand and market supply at the threshold price.

$$Q^D\left(2\sqrt{c^f}\right) = a - 2\sqrt{c^f} = n\sqrt{c^f} = Q^S\left(2\sqrt{c^f}\right) \quad \Rightarrow \quad n = \frac{a}{\sqrt{c^f}} - 2$$

The number of firms must be a non-negative integer. Thus, the number of firms in equilibrium is given by

$$n^* = \max\left\{ \left\lfloor \frac{a}{\sqrt{c^f}} - 2 \right\rfloor, 0 \right\},$$

which is a piecewise-defined function.[1]

[1] The floor function $\lfloor x \rfloor$ gives the greatest integer less than or equal to x.

(b) In the long run, profit per firm in equilibrium must be non-negative; it is strictly positive, if and only if the number of firms that leads to zero profits for each firm is a non-integer greater than 1.

(i) If $a = 120$ and $c^f = 100$, the number of firms in equilibrium is $n^* = 10$, and the equilibrium price is $p^* = 20$. The corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 10$. It follows that profit per firm is $\pi^* = 0$.

(ii) If $a = 120$ and $c^f = 64$, the number of firms in equilibrium is $n^* = 13$, and the equilibrium price is $p^* = 16$. The corresponding equilibrium quantity is $Q^* = 104$, where each firm produces $q^* = 8$. It follows that profit per firm is $\pi^* = 0$.

(iii) If $a = 126$ and $c^f = 100$, the number of firms in equilibrium is $n^* = 10$, and the equilibrium price is $p^* = 21$. The corresponding equilibrium quantity is $Q^* = 105$, where each firm produces $q^* = 10.5$. It follows that profit per firm is $\pi^* = 10.25$.

(c) A firm's total costs plus tax payment are

$$C(q) + tq = \begin{cases} c^f + q^2 + tq, & q > 0 \\ 0, & q = 0. \end{cases}$$

If a firm produces $q > 0$, this quantity must satisfy the condition for a profit maximum $p - t = \mathrm{MC}(q)$, which implies

$$p = 2q + t \quad \Leftrightarrow \quad q = \frac{1}{2}(p - t).$$

In the long run, a firm will produce $q > 0$ according to the above condition if and only if this yields a non-negative profit. Otherwise, the firm will produce $q = 0$. The firm's profit is zero at the break-even quantity, where marginal costs plus tax rate (i.e. marginal tax payment) equal average total costs plus tax rate (i.e. average tax payment).

$$\mathrm{MC}(q) + t = 2q + t = \frac{c^f}{q} + q + t = \mathrm{AC}(q) + t \quad \Rightarrow \quad q = \sqrt{c^f}$$

The corresponding threshold price, which induces the firm to produce the break-even quantity, is

$$p = \mathrm{MC}\left(\sqrt{c^f}\right) + t \quad \Rightarrow \quad p = 2\sqrt{c^f} + t.$$

It follows that individual supply is

$$q(p) = \begin{cases} \frac{1}{2}(p-t), & p \geq 2\sqrt{c^f} + t \\ 0, & p < 2\sqrt{c^f} + t, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} \frac{n}{2}(p-t), & p \geq 2\sqrt{c^f} + t \\ 0, & p < 2\sqrt{c^f} + t. \end{cases}$$

In any competitive equilibrium, market demand equals market supply:

$$Q^D(p) = a - p = \frac{n}{2}(p-t) = Q^S(p).$$

The number of firms that leads to zero profits for each firm equalizes market demand and market supply at the threshold price.

$$Q^D\left(2\sqrt{c^f} + t\right) = a - 2\sqrt{c^f} - t = n\sqrt{c^f} = Q^S\left(2\sqrt{c^f} + t\right)$$

$$\Rightarrow \quad n = \frac{a-t}{\sqrt{c^f}} - 2$$

The number of firms must be a non-negative integer. Thus, the number of firms is equilibrium is

$$n^* = \max\left\{ \left\lfloor \frac{a-t}{\sqrt{c^f}} - 2 \right\rfloor, 0 \right\}.$$

(d) If $a = 126$, $c^f = 100$, and $t = 6$, the number of firms in equilibrium is $n^* = 10$, and the equilibrium price is $p^* = 26$. The corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 10$. It follows that profit per firm is $\pi^* = 0$, tax revenue is $T = 600$, and the welfare loss of taxation is $\text{WL} = 15$.[2]

[2] If $a = 126$, $c^f = 100$, and $t = 0$, the equilibrium number of firms is $n^* = 10$, and output is $Q^* = 105$ (see also (b) (iii)). Thus, the change in tax rate from $t = 0$ to $t = 6$ does not affect the number of firms in equilibrium, but it reduces equilibrium output by 5 units implying a welfare loss of $\text{WL} = \frac{1}{2} \cdot 6 \cdot 5 = 15$. Graphically, the welfare loss is the area of the triangle, where the tax rate is the base and output reduction is the height.

Problem 2-6 (*Competitive Equilibrium*)

If a firm produces $q > 0$, this quantity must satisfy the condition for a profit maximum $p = \text{MC}(q)$, which implies

$$p = 20 + \frac{1}{2}q \quad \Leftrightarrow \quad q = 2p - 40.$$

In the long run, a firm will produce $q > 0$ according to the above condition if and only if this yields a non-negative profit. Otherwise, the firm will produce $q = 0$. The firm's profit is zero at the break-even quantity, where marginal costs equal average total costs.

$$\text{MC}(q) = 20 + \frac{1}{2}q = \frac{25}{q} + 20 + \frac{1}{4}q = \text{AC}(q) \quad \Rightarrow \quad q = 10$$

The corresponding threshold price, which induces the firm to produce the break-even quantity, is

$$p = \text{MC}(10) \quad \Rightarrow \quad p = 25.$$

It follows that individual supply is

$$q(p) = \begin{cases} 2p - 40, & p \geq 25 \\ 0, & p < 25, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} n(2p - 40), & p \geq 25 \\ 0, & p < 25. \end{cases}$$

In equilibrium, market demand equals market supply:

$$Q^D(p) = 125 - p = n(2p - 40) = Q^S(p).$$

Problem 2

The number of firms that leads to zero profits for each firm equalizes market demand and market supply at the threshold price.

$$Q^D(25) = 125 - 25 = n(2 \cdot 25 - 40) = Q^S(25) \quad \Rightarrow \quad n = 10$$

Since this is a non-negative integer, $n^* = 10$ is the number of firms in equilibrium.

$\Rightarrow$ **(B)** is correct.

Problem 3

If the number of firms in equilibrium is $n^* = 10$, the equilibrium price is $p^* = 25$ and the equilibrium quantity is $Q^* = 100$. Then, consumer surplus and producer surplus are given by

$$\text{CS} = \frac{1}{2} \cdot (125 - 25) \cdot 100 = 5{,}000 \quad \text{and} \quad \text{PS} = \frac{1}{2} \cdot (25 - 20) \cdot 100 = 250.$$

$\Rightarrow$ **(B)** is correct.

Problem 4

In the market equilibrium without the price ceiling, the equilibrium price $p^* = 25$ is equal to the threshold price, which implies that each firm produces the break-even quantity $q = 10$ and makes zero profits. Any price ceiling below this threshold price makes it impossible for firms to break even when producing $q > 0$. Therefore, each firm will produce $q = 0$ in the long run, and no surplus will be realized. It follows, that a price ceiling at $p' = 20$ results in a welfare loss equal to total surplus that would be realized in the equilibrium without the price ceiling $\text{TS} = \text{CS} + \text{PS} = 5{,}000 + 250 = 5{,}250$.

$\Rightarrow$ **(D)** is correct.

Problem 5

If the equilibrium price is $p^* = 25$, the introduction of a price floor at $p'' = 20$ has no effect, so that the resulting welfare loss is 0.

$\Rightarrow$ **(A)** is correct.

Problem 6

A lump-sum subsidy $S = 24$ for each firm that produces $q > 0$ effectively lowers quasi-fixed costs, so that a firm's total costs minus the subsidy are

$$C(q) - S = \begin{cases} 1 + 20q + \frac{1}{4}q^2, & q > 0 \\ 0, & q = 0. \end{cases}$$

It follows that market supply is

$$Q^S(p) = \begin{cases} n(2p - 40), & p \geq 21 \\ 0, & p < 21. \end{cases}$$

The number of firms that leads to zero profits for each firm equalizes market demand and market supply at the threshold price.

$$Q^D(21) = 125 - 21 = n(2 \cdot 21 - 40) = Q^S(21) \quad \Rightarrow \quad n^* = 52$$

$\Rightarrow$ **(D)** is correct.