Solution 5: Market Failure

Problem 1 (*Monopoly*)

(a) Inverse market demand for sparkling wine per opera season is

$$p(Q) = 10 - \frac{1}{1{,}000}Q.$$

Thus, the seller's revenue is

$$R(Q) = 10Q - \frac{1}{1{,}000}Q^2.$$

The payment for the exclusive right to sell sparkling wine during one opera season is a quasi-fixed cost c^f. If the firm is awarded the contract, its total costs are

$$C(Q) = c^f + 2Q.$$

As a monopolist, the firm solves the following profit maximization problem.

$$\max_Q \quad \pi(Q) = 10Q - \frac{1}{1{,}000}Q^2 - c^f - 2Q$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 8 - \frac{1}{500}Q = 0$$

yields the monopoly quantity

$$Q^M = 4{,}000.$$

The corresponding monopoly price is $p^M = 6$, and the monopoly profit is $\pi^M = 16{,}000 - c^f$. The firm is only interested in selling sparkling wine at the opera if this yields a non-negative profit. Thus, the firm's maximum willingness to pay for the exclusive right of sale during one opera season is $c^f = 16{,}000$.

(b) If the city of Munich levies a tax on the seller of sparkling wine

(i) at the rate $t = 2$ per unit sold, the firm's profit-maximization problem is

$$\max_{Q} \quad \pi(Q) = 10Q - \frac{1}{1{,}000}Q^2 - c^f - 2Q - \underbrace{2Q}_{tQ}.$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 6 - \frac{1}{500}Q = 0$$

yields the monopoly quantity

$$Q^M = 3{,}000.$$

The corresponding monopoly price is $p^M = 7$, and the monopoly profit after taxes is $\pi^M = 9{,}000 - c^f$. The firm's maximum willingness to pay for the exclusive right of sale during one opera season is $c^f = 9{,}000$.

(ii) at the rate $t = 0.25$ on profit, the firm's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = \underbrace{(1 - 0.25)}_{1-t} \left[10Q - \frac{1}{1{,}000}Q^2 - 2Q - c^f\right].$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 0.75 \left[8 - \frac{1}{500}Q\right] = 0$$

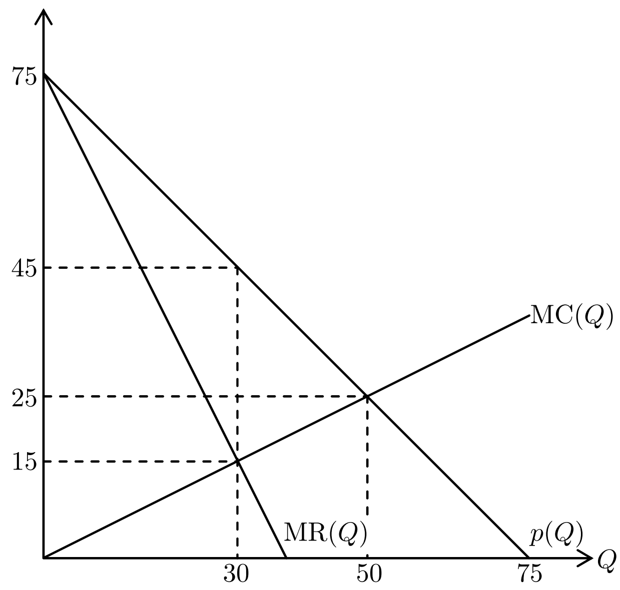
yields the monopoly quantity

$$Q^M = 4{,}000.$$

The corresponding monopoly price is $p^M = 6$, and the monopoly profit after taxes is $\pi^M = 0.75\left(16{,}000 - c^f\right)$. The firm's maximum willingness to pay for the exclusive right of sale during one opera season is $c^f = 16{,}000$.[1]

[1] Note that for $c^f = 16{,}000$, the firm makes zero profits and thus pays no taxes.

Problems 2-5 (*Monopoly*)



Monopoly

The monopolist's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = 75Q - Q^2 - \frac{1}{4}Q^2 - c^f.$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 75 - 2\frac{1}{2}Q = 0$$

yields the monopoly quantity

$$Q^M = 30.$$

The corresponding monopoly price is $p^M = 45$, and the monopoly profit is $\pi^M = 1{,}125 - c^f$. For comparison, the welfare maximizing output for which inverse market demand equals marginal costs is $Q^* = 50$, and the corresponding price is $p^* = 25$.

Problem 2

The monopolist's profit is non-negative if quasi-fixed costs satisfy $c^f \leq 1{,}125$. Thus, the threshold regarding quasi-fixed costs, below which the monopolist's output is $Q > 0$ in the long run is $c^f = 1{,}125$.

$\Rightarrow$ **(C)** is correct.

Problem 3

If fixed costs are $c^f = 625$, the monopolist produces $Q^M = 30$. A welfare loss occurs because the monopoly output Q^M is smaller than the welfare maximizing output Q^*. Hence, for the output difference $Q^* - Q_M$, potential gains from trade exist but are not realized.

The welfare loss is

$$\text{WL} = \frac{1}{2} \cdot \left(Q^* - Q^M\right) \cdot \left(p(Q^M) - \text{MC}(Q^M)\right) = \frac{1}{2} \cdot (50 - 30) \cdot (45 - 15) = 300.$$

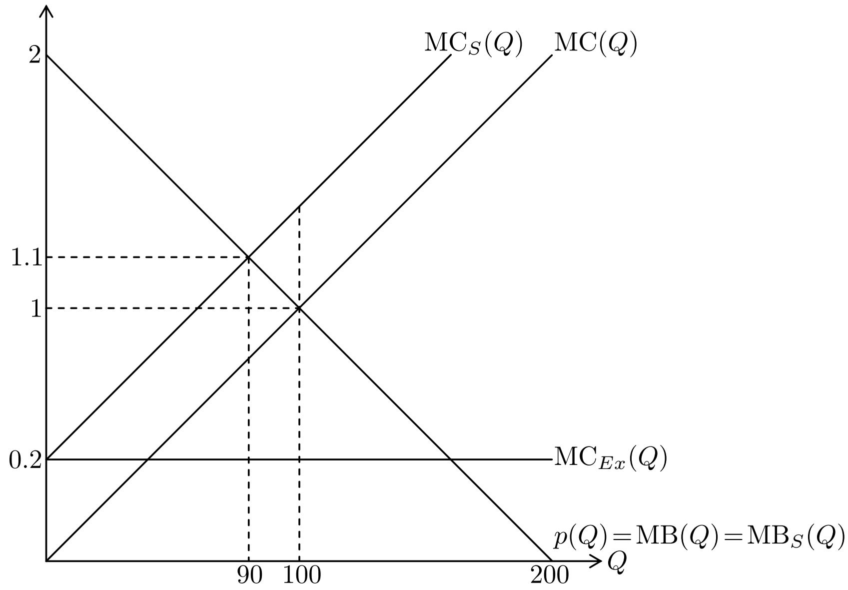
$\Rightarrow$ **(D)** is correct.

Problem 4

If fixed costs are $c^f = 0$, the monopoly output is $Q^M = 30$ and the monopoly price is $p^M = 45$. A price ceiling at $p' = 40$ will induce the monopolist to increase output to $Q^D(p') = 35$. This causes an increase in consumer surplus.

$\Rightarrow$ **(A)** is correct.

Problem 5-8 (*External Effects*)



External Effect

Problem 5

Individual profit maximization implies that the firms do not internalize the negative externalities they impose on each other.

The maximization problem of firm $i \in \{A, B\}$ is

$$\max_{q_i} \quad \pi_i(q_i) = p \cdot q_i - 15 - \frac{1}{100}q_i^2 - \frac{1}{5}q_j.$$

The necessary condition is:

$$\frac{d\pi_i(q_i)}{dq_i} = p - \frac{1}{50}q_i = 0 \quad \Rightarrow \quad p = \underbrace{\frac{1}{50}q_i}_{\text{MC}(q_i)}$$

Rearranging yields the supply of firm $i \in \{A, B\}$.

$$q_i(p) = 50p$$

For two identical firms, market supply is then

$$Q^S(p) = 100p.$$

The equilibrium price equalizes market demand and market supply.

$$Q^D(p^*) = 200 - 100p^* = 100p^* = Q^S(p^*)$$

Hence, the equilibrium price is $p^* = 1$, and the corresponding equilibrium quantity is $Q^* = 100$, where firm $i \in \{A, B\}$ produces $q_i^* = 50$, and profit of firm $i \in \{A, B\}$ is $\pi_i = 0$.

$\Rightarrow$ **(A)** is correct.

Problem 6

Suppose, hypothetically, that the two firms collectively maximize profit (and yet remain price takers). Collective maximization implies that the firms internalize the negative externalities they impose on each other. The resulting equilibrium quantity maximizes welfare as it aligns social marginal benefit to social marginal costs.

The collective maximization problem of firms A and B is

$$\max_{q_A, q_B} \quad \Pi(q_A, q_B) = p \cdot q_A + p \cdot q_B - 15 - \frac{1}{100}q_A^2 - \frac{1}{5}q_B - 15 - \frac{1}{100}q_B^2 - \frac{1}{5}q_A.$$

The necessary conditions are:

$$\frac{\partial \Pi(q_A, q_B)}{\partial q_A} = p - \frac{1}{50}q_A - \frac{1}{5} = 0 \quad \Rightarrow \quad p = \underbrace{\frac{1}{50}q_A}_{\text{MC}(q_A)} + \underbrace{\frac{1}{5}}_{\text{MC}_{Ex}(q_A)}$$

$$\frac{\partial \Pi(q_A, q_B)}{\partial q_B} = p - \frac{1}{50}q_B - \frac{1}{5} = 0 \quad \Rightarrow \quad p = \underbrace{\frac{1}{50}q_B}_{\text{MC}(q_B)} + \underbrace{\frac{1}{5}}_{\text{MC}_{Ex}(q_B)}$$

Rearranging yields the supply of firm $i \in \{A, B\}$.

$$q_i(p) = 50p - 10$$

For two identical firms, market supply is then

$$Q^S(p) = 100p - 20.$$

The equilibrium price equalizes market demand and market supply.

$$Q^D(p^{**}) = 200 - 100p^{**} = 100p^{**} - 20 = Q^S(p^{**})$$

Hence, the equilibrium price is $p^{**} = 1.1$, and the corresponding equilibrium quantity is $Q^{**} = 90$, where firm $i \in \{A, B\}$ produces $q_i^{**} = 45$, and profit of firm $i \in \{A, B\}$ is $\pi_i = 5.25$.

Accordingly, the welfare maximizing total quantity is $Q_{Opt} = 90$.

$\Rightarrow$ **(C)** is correct.

Problem 7

A welfare loss occurs because the equilibrium quantity Q^*, resulting from individual profit maximization, exceeds the welfare maximizing quantity Q_{Opt}. Hence, for the exceeding quantity $Q^* - Q_{Opt}$, social marginal costs exceed social marginal benefits.

The welfare loss is

$$\text{WL} = \frac{1}{2} \cdot (Q^* - Q_E) \cdot \text{MC}_{Ex}(Q^*) = \frac{1}{2} \cdot (100 - 90) \cdot \frac{1}{5} = 1.$$

$\Rightarrow$ **(B)** is correct.

Problem 8

Since the rate of this (Pigouvian) tax equals external marginal costs, individual profit maximization implies that firms effectively internalize the negative externalities they impose on each other. The resulting equilibrium quantity maximizes welfare as it aligns social marginal benefit to social marginal costs.

The maximization problem of firm $i \in \{A, B\}$ is

$$\max_{q_i} \quad \pi_i(q_i) = p \cdot q_i - 15 + S - \frac{1}{100} q_i^2 - \frac{1}{5} q_j - \underbrace{\frac{1}{5} q_i}_{tq_i}.$$

The necessary condition is:

$$\frac{d\pi_i(q_i)}{dq_i} = p - \frac{1}{50} q_i - \frac{1}{5} = 0 \quad \Rightarrow \quad p = \underbrace{\frac{1}{50} q_i}_{\text{MC}(q_i)} + \underbrace{\frac{1}{5}}_{t}$$

Rearranging yields the supply of firm $i \in \{A, B\}$

$$q_i(p) = 50p - 10.$$

For two firms, market supply is then

$$Q^S(p) = 100p - 20.$$

The equilibrium price equalizes market demand and market supply.

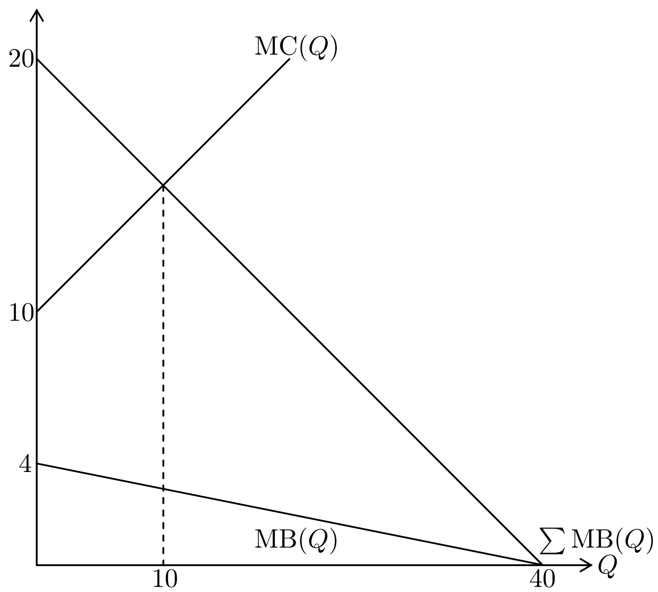
$$Q^D(p^*) = 200 - 100p^* = 100p^* - 20 = Q^S(p^*)$$

Hence, the equilibrium price is $p^* = 1.1$, and the corresponding equilibrium quantity is $Q^* = 90$, where firm $i \in \{A, B\}$ produces $q_i^* = 45$, and profit of firm $i \in \{A, B\}$ is $\pi_i = S - 3.75$.

It follows that firms make zero profits if the lump-sum subsidy is $S = 3.75$.

$\Rightarrow$ **(A)** is correct.

Problem 9-10 (*Public Goods*)



Public Good

Problem 9

Suppose, hypothetically, that the five individuals collectively provided the public good in order to maximize total surplus, i.e. welfare.

The necessary condition (known as the Samuelson condition) is:

$$\sum \text{MB}(Q) = \text{MC}(Q)$$
$$5 \cdot \text{MB}(Q) = \text{MC}(Q)$$

Substituting yields:

$$20 - \frac{1}{2}Q = 10 + \frac{1}{2}Q \quad \Rightarrow \quad Q_{Opt} = 10$$

$\Rightarrow$ **(B)** is correct.

Problem 10

Individual provision of the public good implies that each individual maximizes individual surplus.

The necessary condition for an interior solution of the individual maximization problem is:

$$\text{MB}(Q) = \text{MC}(Q)$$

Substituting shows that the necessary condition is never satisfied since

$$4 - \frac{1}{10}Q < 10 + \frac{1}{2}Q \quad \forall \quad Q \geq 0.$$

Hence, no individual contributes to the provision of the public good. Consequently, the resulting quantity is $Q = 0$ (corner solution).

A welfare loss occurs because individual maximization implies an underprovision of the public good. The public good is not provided at all, although, the sum of marginal benefits exceeds marginal costs up to the welfare maximizing quantity Q_{Opt}.

The welfare loss is

$$\text{WL} = \frac{1}{2} \cdot Q_{Opt} \cdot (5 \cdot \text{MB}(0) - \text{MC}(0)) = \frac{1}{2} \cdot 10 \cdot (20 - 10) = 50.$$

$\Rightarrow$ **(C)** is correct.